Structural FECM: Cointegration in large-scale structural FAVAR models

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Abstract

Starting from the dynamic factor model for non-stationary data we derive the factor-augmented error correction model (FECM) and, by generalizing the Granger representation theorem, its moving-average representation. The latter is used for the identification of structural shocks and their propagation mechanism. Besides discussing contemporaneous restrictions along the lines of Bernanke et al. (2005), we show how to implement classical identification schemes based on long-run restrictions in the case of large panels. The importance of the error-correction mechanism for impulse response analysis is analysed by means of both empirical examples and simulation experiments. Our results show that the bias in estimated impulse responses in a FAVAR model is positively related to the strength of the error-correction mechanism and the cross-section dimension of the panel. We observe empirically in a large panel of US data that these features have a substational effect on the responses of several variables to the identified real shock.

Keywords: Dynamic Factor Models, Cointegration, Structural Analysis, Factor-augmented Error Correction Models, FAVAR

JEL-Codes: C32, E17

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1 Introduction

The factor-augmented error-correction model (FECM) was introduced by Banerjee and Marcellino (2009). Focusing on modelling small-scale models they proposed using the factors extracted from large non-stationary panels to proxy for missing cointegration relations. Banerjee, Marcellino and Masten (2013) showed that the FECM in several instances outperforms both the FAVAR and the standard error-correction model in forecasting precision.

This paper focuses on the implications for structural analysis of modelling cointegration in large systems of nonstationary variables. Starting from the seminal work by Bernanke et al. (2005) the literature provides many examples of how the factor-augmented VAR model (FAVAR) can be used for macroeconomic modelling, and the analysis of structural shocks in particular (see Stock and Watson (2005) for a detailed discussion). In estimating the FAVAR model, the original nonstationary data are typically transformed in a way to render them stationary. In this way the FAVAR neglects the potential cointegration possibilities between the variables in the panel and the factors as analysed by Bai (2004). With the FECM we introduce cointegration into the structural modelling of large systems of variables. In this respect the FECM is a nesting model for both the FAVAR and the standard small-scale error-correction model.

In this paper we show how, for the purposes of modelling large systems of variables, the FECM can be derived from a dynamic factor model for nonstationary data, where the starting point for our analysis is the factor model studied by Bai (2004). Bai's asymptotic results can also be applied in our context, when a mixture of I(1) and I(0) factors is allowed, for both the identification of the factor spaces and the estimation of the factors.

With the objective of discussing identification of structural shocks we also derive the moving-average representation of the FECM. Using the latter we provide a generalization of the Granger representation theorem to nonstationary panels of cointegrated variables.

The discussion of identification schemes for structural shocks is straightforward. We show how the FECM can be used to identify monetary policy shocks as in Bernanke et al. (2005). More importantly, our paper provides the first analysis of the long-run scheme for identification of structural shocks in nonstationary panels. Forni, Giannone, Lippi and Reichlin (2009) provide an empirical illustration of the stochastic trends analysis of King, Plosser, Stock and Watson (1991) in the context of large stationary panels. Eickmeier (2009) works with a nonstationary panel and the identification of structural shocks with sign restrictions. By considering long-run restrictions in nonstationary panels our paper fills in a gap in the literature.

Two simulation experiments are presented to justify the applicability of the Bai famework for samples with finite N and T. The first shows that the principal component-based estimator efficiently estimates the spaces spanned by both the I(1) and I(0) factors. FECM estimates therefore have good properties also in finite samples.

The second simulation experiment tests our intuition that the importance of our modelling and identification scheme will depend on the strength of the error correction mechanism. Moreover, as shown by a simple analytical example presented in the paper, the FECM can to some extent be approximated by the FAVAR with sufficiently large lag length, estimation uncertainty may also play a role. These conjectures are tested by means of an empirically motivated simulation experiment which confirms that the bias in estimated impulse responses from ignoring the long-run in a FAVAR model is positively related to the strength of the error-correction mechanism and the cross-section dimension of the panel.

When assessing the empirical properties of the FECM with respect to the FAVAR we focus on the effects of including the error-correction terms into the model on the structural impulse responses in two applications where, for comparability, we use the dataset of Bernanke et al. (2005) for the US economy. The first empirical application concerns the identification of a monetary policy shock in a FECM with contemporaneous restrictions, while the second is a demonstration of our proposed long-run identification scheme for the analysis of structural stochastic trends. The latter are especially important, because these shocks account for the largest shares of overall panel variability and thus make the effects of omitting the error-correction terms in the FAVAR most pronounced. Indeed, with long-run restrictions, our empirical results reveal quite considerable statistically significant differences between the FAVAR and the FECM.

The paper is structured as follows. Section 2 provides the representation of the FECM and discusses its estimation. Section 3 presents a simple analytical example and show how the impulse response analysis differs between the FECM and the FAVAR. Section 4 derives the moving-average representation of the FECM and discusses the identification of structural shocks. Section 5 contains the two simulation experiments described above to address, first, the finite sample properties of the FECM and, second, to elaborate upon the differences in the impulse response analysis obtained from estimating a FECM instead of a FAVAR. Section 6 contains the empirical examples and Section 7 concludes.

2 Factor-augmented error-correction model

The factor-augmented error-correction model (FECM) is nested within the dynamic factor model for I(1) data developed by Bai (2004) as it allows for both I(1) and I(0) factors, which is also the starting point of our analysis. This allows us to distinguish between common stochastic trends and stationary drivers of all variables. In order to estimate all the parameters of the FECM, we need to strengthen one aspect of Bai's (2004) model to require a strict dynamic factor model even further. This restriction, which is a specialization of the Bai assumptions, nevertheless leaves all of his results directly applicable to our model as verified by the simulation experiments in Section 5 of the paper.

2.1 Representation of the FECM

Consider the the following dynamic factor model (DFM) for I(1) data

$$
\begin{aligned}
X_{it} &= \sum_{j=0}^{p} \lambda_{ij} F_{t-j} + \sum_{j=0}^{m} \phi_{il} c_{t-l} + \varepsilon_{it} \\
&= \lambda_i(L) F_t + \phi_i(L) c_t + \varepsilon_{it},
\end{aligned} \tag{1}
$$

where $i = 1, ..., N,\ t = 1, ..., T,\ \ F_t$ is a r_1-dimensional vector of random walks and c_t is a r_2-dimensional vector of I(0) factors and ε_{it} is a zero-mean idiosyncratic component. $\lambda_i\,(L)$and $\varphi_i\,(L)$ are lag polynomials of orders p and m respectively. To derive the limiting distribution of estimators of F_t and c_t respectively, p and m are assumed to be finite. We assume that $\lambda_i(L) = \sum_{j=0}^{\infty} a_{ij} L^j$ and $\phi_i(L) = \sum_{j=0}^{\infty} b_{ij} L^j$ with $\sum_{j=0}^{\infty} j\,|a_{ij}|$ and $\sum_{j=0}^{\infty} j\,|b_{ij}|$ finite. Common components $\lambda_i(L)F_t$ and $\phi_i(L)c_t$ are well defined as it is assumed that $F_t = c_t = 0$ for $t < 0$. The loading matrices λ_{ij} and ϕ_{ij} are either deterministic or stochastic and satisfy the following restrictions. For $\lambda_i = \lambda_i(1)$ and $\phi_i = \phi_i(1)$ we have $E\,\|\lambda_i\|^4 \leq M < \infty$, $E\,\|\phi_i\|^4 \leq M < \infty$, and $1/N \sum_{i=0}^{N} \lambda_i \lambda_i'$ $1/N \sum_{i=0}^{N} \phi_i \phi_i'$ converge in probability to positive definite matrices. Furthermore, we assume that $E\,(\lambda_{ij}\varepsilon_{is}) = E\,(\phi_{ij}\varepsilon_{is}) = 0$ for all i, j and s.

In our treatment of the idiosyncratic component ε_{it} we are more restrictive than Bai (2004). ε_{it} is allowed to be serially correlated $\varepsilon_{it} = \gamma_i(L)\varepsilon_{it-1} + v_{it}$ with the roots of $\gamma_i(L)$ lying inside the unit disc. Because this assumption holds for all i, X_{it} and F_t cointegrate. Finally, to avoid the curse of dimensionality in estimating the FECM, we assume (1) to be a strict factor model: $E\,(\varepsilon_{it}, \varepsilon_{js}) = 0$ for all i, j, t and s, $i \neq j$.[1]

To derive the FECM and discuss further assumptions upon the model that ensure consistent estimation of the model's components, it is convenient to write first the DFM in static form. To this end we follow Bai (2004) and define the matrices

$$\tilde{\lambda}_{ik} = \lambda_{ik} + \lambda_{ik+1} + ... + \lambda_{ip},\ \ k = 0, ..., p$$

Let us in addition define the matrix

$$\tilde{\Phi}_i = [\phi_{i0}, ..., \phi_{im}]'$$

Then we can get a static representation of the DFM which has the I(1) factors isolated from the I(0) factors:

$$X_{it} = \Lambda_i F_t + \Phi_i G_t + \varepsilon_{it} \tag{2}$$

[1] On a dataset similar to ours, Stock and Watson (2005) show that the strict factor model assumption is generally rejected but is of limited quantitative importance.

where

$$
\begin{aligned}
\Lambda_i &= \tilde{\lambda}_{i0} \\
\Phi_i &= \left[-\tilde{\lambda}_{i1}, \ldots, -\tilde{\lambda}_{ip}, \tilde{\Phi}_i\right] \\
G_t &= \left[c_t', c_{t-1}', ..., c_{t-m}', \Delta F_t', ..., \Delta F_{t-p}'\right]'
\end{aligned}
$$

Introducing for convenience the notation $\Psi_i = [\Lambda_i', \Phi_i']'$, the following assumptions are needed for consistent estimation of both the I(1) and I(0) factors: $E\left\|\Psi_i\right\|^4 \leq M < \infty$ and $1/N \sum_{i=0}^{N} \Psi_i \Psi_i'$ converges to a $(r_1(p+2)+r_2(m+1)) \times (r_1(p+2)+r_2(m+1))$ positive-definite matrix.

Grouping across the N variables we have

$$
X_t = \Lambda F_t + \Phi G_t + \varepsilon_t \tag{3}
$$

where $X_t = [X_{1t}, ..., X_{Nt}]'$, $\Lambda = [\Lambda_1, ..., \Lambda_N]'$, $\Phi = [\Phi_1, ..., \Phi_N]'$ and $\varepsilon_t = [\varepsilon_{1t}, ..., \varepsilon_{Nt}]'$.

As noted above, the idiosyncratic component in (3) is serially correlated. This serial correlation can be eliminated from the error process by premultiplying (2) by

$$
I - \Gamma\left(L\right) L
$$

where

$$
\Gamma\left(L\right) = \begin{bmatrix} \gamma_1\left(L\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_N\left(L\right) \end{bmatrix}.
$$

Following this transformation, we obtain

$$
X_t = \left(I - \Gamma\left(L\right) L\right) \Lambda F_t + \left(I - \Gamma\left(L\right) L\right) \Phi G_t + \Gamma\left(L\right) X_{t-1} + v_t.
$$

Note that $\Gamma\left(L\right)$ can be conveniently factorized as

$$
\Gamma(L) = \Gamma(1) - \Gamma_1(L)(1-L),
$$

which allows us to rewrite the previous expression as

$$
\begin{aligned}
X_t = {} & \Lambda F_t + \Phi G_t - (\Gamma(1) - \Gamma_1(L)(1-L))(\Lambda F_{t-1} + \Phi G_{t-1}) \\
& + (\Gamma(1) - \Gamma_1(L)(1-L)) X_{t-1} + v_t.
\end{aligned} \tag{4}
$$

This can be further expanded as

$$
\begin{aligned}
X_t = {} & \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\
& + \Gamma_1(L)\Lambda\Phi\Delta G_{t-1} + \Gamma(1) X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t
\end{aligned} \tag{5}
$$

or

$$\Delta X_t = \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\ + \Gamma_1(L)\Phi\Delta G_{t-1} - (I - \Gamma(1))X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t \tag{6}$$

The ECM form of the DFM or the factor-augmented error-correction model - the FECM - then follows directly as

$$\Delta X_t = \underbrace{-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})}_{\text{Omitted in the FAVAR}} + \Lambda\Delta F_t + \Gamma_1(L)\Lambda\Delta F_{t-1} \\ + \Phi G_t - \Gamma(1)\Phi G_{t-1} + \Gamma_1(L)\Phi\Delta G_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t. \tag{7}$$

Equation (7) is a representation of the DFM for data that are stationary, corresponding to standard applications of the FAVAR model, which suitably transform nonstationary data. It contains the error-correction term, $-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})$, which is omitted in the FAVAR model.

Note that it follows from (3) that

$$X_{t-1} - \Lambda F_{t-1} = \Phi G_{t-1} + \varepsilon_{t-1},$$

such that it would appear at first sight that the omitted error-correction term in the FAVAR could be approximated by including additional lags of the I(0) factors. However, by substituting the previous expression into (7) and symplifying we get

$$\Delta X_t = \Lambda\Delta F_t + \Phi\Delta G_t + \Delta\varepsilon_t, \tag{8}$$

which contains a non-invertible MA component. Hence, whenever we deal with I(1) data, and many macroeconomic series exhibit this feature, the standard FAVAR model produces biased results unless we use an infinite number of factors as regressors, or account explicitly for the MA structure of the error-process.[2] The analytical example in the next section elaborates this point further.

Our empirical and simulation analyses below show that omission of the ECM term in the FAVAR may potentially have an important impact on the results obtained in typical macroeconomic applications.

To complete the model, we assume that the nonstationary factors follow a vector random walk process

$$F_t = F_{t-1} + \varepsilon_t^F, \tag{9}$$

[2] Our empirical application below is based on the dataset used by Bernanke et al., 2005). They treat 77 out of 120 series as I(1).

while the stationary factors are represented by

$$c_t = \rho c_{t-1} + \varepsilon_t^c, \tag{10}$$

where ρ is a diagonal matrix with values on the diagonal in absolute term strictly less than one. ε_t^F and ε_t^c are independent of λ_{ij}, ϕ_{ij} and ε_{it}. As in Bai (2004), it should be noted that the error processes ε_t^F and ε_t^c need not necessarily be *i.i.d.* They are allowed to be serially and cross correlated and jointly follow a stable vector process:

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = A(L) \begin{bmatrix} \varepsilon_{t-1}^F \\ \varepsilon_{t-1}^c \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{11}$$

where u_t and w_t are zero-mean white-noise innovations to dynamic nonstationary and stationary factors, respectively. Under the stability assumption, we can express these as

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = [I - A(L)L]^{-1} \begin{bmatrix} u_t \\ w_t \end{bmatrix} \tag{12}$$

Note that, under these assumptions, we have $E\left\|\varepsilon_t^F\right\|^4 \leq M < \infty$, which implies that $\sum_{t=1}^T F_t F_t'$ converges at rate T^2, while $\sum_{t=1}^T G_t G_t'$ converges at the standard rate T. The cross-product matrices $\sum_{t=1}^T F_t G_t'$ and $\sum_{t=1}^T G_t' F_t$ converge at rate $T^{3/2}$. At these rates the matrix composed of these four elements jointly converge to a positive definite matrix, allowing us to apply Bai's (2004) consistency results on factor estimation using principal components.

Using (9), (10) and (12) we can write the VAR of dynamic factors as

$$\begin{aligned} \begin{bmatrix} F_t \\ c_t \end{bmatrix} &= \left[\begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} + A(L) \right] \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} - A(L) \begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} F_{t-2} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix} \\ &= C(L) \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \end{aligned} \tag{13}$$

where the parameter restrictions imply that $C(1)$ is a block-diagonal matrix with block sizes corresponding to the partition between F_t and c_t.

The FECM is specified in terms of static factors F and G, which calls for a corresponding VAR specification. Using the definition of G_t and (13) it is straightforward to get the following representation

$$
\begin{bmatrix}
I & 0 & \cdots & & & \cdots & 0 \\
0 & I & \cdots & & & \cdots & 0 \\
\vdots & & & & & & \vdots \\
\vdots & & & & & & \vdots \\
0 & \cdots & I & 0 & & \cdots & 0 \\
-I & \cdots & 0 & I & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & I & \cdots & 0 \\
\vdots & & & & & & \vdots \\
0 & \cdots & & \cdots & & \cdots & I
\end{bmatrix}
\begin{bmatrix}
F_t \\ c_t \\ c_{t-1} \\ \vdots \\ c_{t-m} \\ \Delta F_t \\ \Delta F_{t-1} \\ \vdots \\ \Delta F_{t-p}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
C_{11}(L) & C_{12}(L) & 0 & \cdots & & & \cdots & 0 \\
C_{21}(L) & C_{22}(L) & 0 & \cdots & & & \cdots & 0 \\
0 & I & 0 & \cdots & & & \cdots & 0 \\
\vdots & & \cdots & & \cdots & & & \vdots \\
0 & \cdots & \cdots & I & 0 & & \cdots & 0 \\
-I & \cdots & & \cdots & & & \cdots & 0 \\
0 & & \cdots & & I & & \cdots & 0 \\
\vdots & & & & & & & \vdots \\
0 & \cdots & & \cdots & & & I & 0
\end{bmatrix}
\begin{bmatrix}
F_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \vdots \\ c_{t-m-1} \\ \Delta F_{t-1} \\ \Delta F_{t-2} \\ \vdots \\ \Delta F_{t-p-1}
\end{bmatrix}
+
\begin{bmatrix}
I & 0 \\ 0 & I \\ 0 & 0 \\ & \\ \vdots & \vdots \\ & \\ \vdots & \vdots \\ & \\ 0 & 0
\end{bmatrix}
\begin{bmatrix}
w_t \\ u_t
\end{bmatrix}
\qquad (14)
$$

Using the definition of G_t, the VAR for the static factors, and premultplying the whole expression by the inverse of the initial matrix in (14), the factor VAR can be more compactly written as

$$
\begin{bmatrix} F_t \\ G_t \end{bmatrix} = \begin{bmatrix} M_{11}(L) & M_{12}(L) \\ M_{21}(L) & M_{22}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ G_{t-1} \end{bmatrix} + Q \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \qquad (15)
$$

where the $(r_1(p+2) + r_2(m+1)) \times (r_1 + r_2)$ matrix Q accounts for dynamic singularity of G_t. This is due to the fact that the dimension of the vector process w_t is r_2, which is smaller than or equal to $r_1(p+2) + r_2(m+1)$, the dimension of G_t. Note that it follows from block-diagonality of $C(1)$ (see (13)) that $M_{21}(1) = 0$, such that F_t is I(1) and G_t is I(0).

2.2 Estimation of the FECM

As discussed in the previous section, the model is consistent with the specification analyzed by Bai (2004) that accommodates the presence of I(0) factors along with I(1) factors in the factor model. This allows us to use the same set of underlying assumptions and invoke Bai's (2004) results on asymptotic properties of the estimated process.

The number of I(1) factors r_1 can be consistently estimated using the criteria developed by Bai (2004) applied to data in levels. The overall number of static factors $r_1(p+2) + r_2(m+1)$ can be estimated using the criteria of Bai and Ng (2002) applied to the data in differences.

The space spanned by the factors can be consistently estimated using principal components. The estimator of F_t are the eigenvectors corresponding to the largest r_1 eigenvalues of XX' normalized such that $\widetilde{F}'\widetilde{F}/T^2 = I$. The stationary factors G_t can be estimated as the eignevectors corresponding to the next q largest eigenvalues normalized such that $\widetilde{G}'\widetilde{G}/T = I$ (Bai, 2004). Corresponding estimators of the loadings to I(1) factors are then $\widetilde{\Lambda} = X'\widetilde{F}/T^2$, and those to the I(0) factors $\widetilde{\Phi} = X'\widetilde{G}/T$.[3]

Using the estimated factors and loadings, the estimates of the common components are $\widetilde{\Lambda}\widetilde{F}_t$, $\widetilde{\Phi}\widetilde{G}_t$, $\widetilde{\Lambda}\Delta\widetilde{F}_t$ and $\widetilde{\Phi}\Delta\widetilde{G}_t$, while for the cointegration relations it is $X_{t-1} - \widetilde{\Lambda}\widetilde{F}_{t-1}$.Finally, the estimated common components and cointegrating relations can be used in (7) to estimate the remaining parameters of the FECM by OLS, equation by equation due to the strict-factor-model assumption. Replacing the true factors and their loadings with their estimated counterparts is permitted under the assumptions discussed above and in Bai (2004) (see Bai,2004, Lemma 3, p. 148) so that we do not have a problem with generated regressors.[4] The theoretical results are verified by means of the simulation experiments reported below.

3 Impulse response analysis in the FECM and FAVAR - an analytical illustration

We illustrate analytically the computation of structural responses using the FECM rather than the FAVAR with a simple but comprehensive example. The example may easily be seen to be a special case of the general specification introduced in the previous section, obtained by restricting the dimension of the factor space and of the variables of interest studied.

We suppose that the large information set available can be summarized by one $I(1)$ common factor, f, and that the econometrician is particularly interested in the response of one of the many variables, x_1, and that she can choose any of the three following models. First, a FECM, where the explanatory variables of the FAVAR are augmented with a term representing the (lagged) deviation from the long run equilibrium of x_1 and f. Second, a FAVAR model where the change in x_1 (Δx_1) is explained by an infinite number of its own lags and by lags of the change in f. And, third, the same model but with a finite number of lags. We want to compare the differences in IRFs resulting from the three models.

[3] In a similar model to ours, Choi (2011) analyzes the generalized principal components estimator that offers some efficiency gains over the classic principal components estimator. Simulation evidence presented below, however, shows that Bai's estimator performs very well already at small sample sizes. For this reason we stick to the standard principal components estimator in this paper.

[4] These assumptions are essentially on (1) the common factor structure of the data, (2) heterogeneous loadings with finite fourth moments, (3) mutual orthogonality between u_t, w_t, ε_{it}, λ_{it} and ϕ_{it}, and (4) weak dependence of idiosyncratic errors.

To start with, let us consider a system consisting of the two variables x_1 and x_2 and of one factor f. The factor follows a random walk process,

$$f_t = f_{t-1} + \varepsilon_t, \tag{16}$$

where ε_t is a structural shock and we are interested in the dynamic response to this shock. The factor loads directly on x_2,

$$x_{2t} = f_t + u_t, \tag{17}$$

while the process for x_1 is given in ECM form as

$$\Delta x_{1t} = \alpha\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma\Delta f_{t-1} + v_t, \quad \alpha < 0. \tag{18}$$

or

$$\Delta x_{1t} = \alpha\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma\varepsilon_{t-1} + v_t. \quad \alpha < 0 \tag{19}$$

Here the processes ε_t and v_t are assumed $i.i.d.(0, I_N)$, while u_t is allowed to have a moving average structure, i.e. $u_t = u_t^* / \left(1 - \eta L\right)$, $|\eta| < 1$ and u_t^* is $i.i.d.(0, \sigma_{u*}^2)$ Hence, the DGP is a FECM.

Note that the moving-average representation of x_{1t} can be written as

$$\begin{aligned}
x_{1t} &= (1+\alpha)^h x_{1t-h} \\
&\quad + (1+\alpha)^{h-1}(-\alpha\beta(\varepsilon_{t-h} + \varepsilon_{t-h-1} + ... + \varepsilon_{-h}) + +\gamma\varepsilon_{t-h} + v_{t-h+1}) \\
&\quad + (1+\alpha)^{h-2}(-\alpha\beta(\varepsilon_{t-h+1} + \varepsilon_{t-h} + ... + \varepsilon_{-h+1}) + \gamma\varepsilon_{t-h+1} + v_{t-h+2}) \\
&\quad \vdots \\
&\quad - (1+\alpha)(\alpha\beta(\varepsilon_{t-1} + \varepsilon_{t-2} + ... + \varepsilon_1) + \gamma\varepsilon_{t-1} + v_t).
\end{aligned}$$

Based on this, the impulse response function takes the following form:

$$\frac{\partial\Delta x_{1t+h}}{\partial\varepsilon_t} = \frac{\partial x_{1t+h}}{\partial\varepsilon_t} - \frac{\partial x_{1t+h-1}}{\partial\varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

The FECM representation of x_1 can also be written as a FAVAR. In fact, since the error-correction term $x_{1t} - \beta f_t$ evolves as

$$\begin{aligned}
x_{1t} - \beta f_t &= \left(\alpha + 1\right)\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma\Delta f_{t-1} + v_t - \beta\varepsilon_t \\
&= \frac{\gamma\Delta f_{t-1}}{1 - \left(\alpha + 1\right)L} + \frac{v_t - \beta\varepsilon_t}{1 - \left(\alpha + 1\right)L},
\end{aligned}$$

we can re-write equation (18) as

$$\Delta x_{1t} = \gamma\Delta f_{t-1} + \frac{\alpha\gamma\Delta f_{t-2}}{1 - \left(\alpha + 1\right)L} + v_t + \frac{\alpha\left(v_{t-1} - \beta\varepsilon_{t-1}\right)}{1 - \left(\alpha + 1\right)L}, \tag{20}$$

which is a FAVAR of infinite order. The corresponding moving-average representation then follows directly as

$$\Delta x_{1t} = \gamma\varepsilon_{t-1} + \frac{\alpha\gamma\varepsilon_{t-2}}{1-(\alpha+1)L} + v_t + \frac{\alpha(v_{t-1}-\beta\varepsilon_{t-1})}{1-(\alpha+1)L}. \tag{21}$$

This implies that the impulse responses of the infinite-order FAVAR model would be

$$\frac{\partial\Delta x_{1t+h}}{\partial\varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

We therefore see that only using a FAVAR with an infinite number of lags allows us to recover the same IRFs as in the FECM. However, in practice, a short lag length is used in the FAVAR, so that the resulting responses will be different from those from the FECM, the more so the poorer the finite lag approximation is to the infinite order FAVAR. A simulation experiment presented later on, whose design is based on a frequently-used panel of US macroeconomic data, reveals that the differences in the impulse responses obtained by the FECM and the FAVAR may be significant.

4 Moving-average representation of the FECM and the Structural FECM

The identification of structural shocks in VAR models usually rests on imposing restrictions upon the parameters of the moving-average representation of the VAR. For vector-error correction models, the derivation of the moving-average representation uses the Granger representation theorem (see, e.g., Johansen, 1995). The FECM is a generalization of error-correction models to large dynamic panels. For this reason, we first provide a generalization of the Granger representation theorem for nonstationary panels that exhibit cointegration. Then we discuss shock identification.

4.1 The MA representation of the FECM

We start with, we have:

Assumption 1 $\omega = [(I_{r_1} - M_{11}^*(1))]^{-1}$ *is an invertible matrix.*

This assumption implies that X_{it} can be at most I(1) and rules out the possibility of X_{it} being and I(2) process, which would results in singular ω.

Theorem 1 (Granger representation for the FECM) *Under assumption A.1 and given the error-correction representation of the dynamic factor model (7), the moving-average* *representation of the factor-augmented error-correction model is*

$$\begin{bmatrix} X_t \\ F_t \\ G_t \end{bmatrix} = \begin{bmatrix} \Lambda \\ I_{r_1} \\ 0_{r_2 \times r_1} \end{bmatrix} \omega \sum_{i=1}^{t} u_i + C_1(L) \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix}. \tag{22}$$

Proof. The factor VAR given by (15) contains exactly r_1 unit roots pertaining to F_t. [5]. (15) can then be rewritten in differenced form as

$$\begin{bmatrix} \Delta F_t \\ \Delta G_t \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_M \end{bmatrix} \begin{bmatrix} 0 & I_{r_2} \end{bmatrix} \begin{bmatrix} F_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} M_{11}^*(L) & M_{12}^*(L) \\ M_{21}^*(L) & M_{22}^*(L) \end{bmatrix} \begin{bmatrix} \Delta F_{t-1} \\ \Delta G_{t-1} \end{bmatrix} + Q \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{23}$$

where the coefficient matrices of the matrix polynomials $M_{ij}^*(L)$ are defined from the coefficient matrices as (15) as (assuming that the polynomial $C(L)$ in (13) is of order n)

$$M_{ijl}^* = -(M_{ijl+1} + M_{ijn}), \quad l = 1, \ldots, n. \tag{24}$$

Furthermore, (7) can be rewritten as

$$\begin{aligned} \Delta X_t = {} & \tilde{\alpha} \left(X_{t-1} - \Lambda F_{t-1} - \Phi G_{t-1} \right) + \Lambda \Delta F_t + \Phi \Delta G_t \\ & + \Gamma_1 (L) \left(\Lambda \Delta F_{t-1} + \Phi \Delta G_{t-1} \right) - \Gamma_1 (L) \Delta X_{t-1} + v_t, \end{aligned} \tag{25}$$

where $\tilde{\alpha} = -(I - \Gamma(1))$. Then we can stack the equations for ΔX_t and the factors into a single system of equations as

$$\begin{aligned} \begin{bmatrix} \Delta X_t \\ \Delta F_t \\ \Delta G_t \end{bmatrix} = {} & \alpha\beta\prime \begin{bmatrix} X_{t-1} \\ F_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} -\Gamma_1(L) & B_1(L) & B_2(L) \\ 0 & M_{11}^*(L) & M_{12}^*(L) \\ 0 & M_{21}^*(L) & M_{22}^*(L) \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \Delta F_{t-1} \\ \Delta G_{t-1} \end{bmatrix} \\ & + \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix} \end{aligned} \tag{26}$$

where $B_1(L) = \Lambda M_{11}^*(l) + \Phi M_{21}^*(L) + \Gamma_1(L)\Phi$ and $B_2(L) = \Phi M_{22}^*(L) + \Lambda M_{12}^*(L) + \Gamma_1(L)\Lambda$ and

$$\alpha = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & 0 \\ 0 & \alpha_M \end{bmatrix} \textit{ and } \beta' = \begin{bmatrix} I & -\Lambda & -\Phi \\ 0 & 0 & I \end{bmatrix}.$$

We can observe that (26) has a structure similar to a standard ECM model with some restrictions imposed and conforms with the assumptions of the Johansen's version of the

[5] Cointegration among F_t is ruled out as we can always include the stationary linear combinations of F_t in G_t. See also Bai (2004)

Granger representation theorem. In particular

$$\beta_\perp = \left[\Lambda', \Phi', I_{r_1+r_2}\right]' \ , \ \alpha_\perp = \begin{bmatrix} \mathbf{0}_{(N)\times r_1} \\ \mathbf{I}_{r_1} \\ \mathbf{0}_{(r_2)\times r_1} \end{bmatrix} \ , \ \Xi = I_{N+r_1} - \begin{bmatrix} -\Gamma_1(1) & B_1(1) & B_2(1) \\ 0 & M_{11}^*(1) & M_{12}^*(1) \\ 0 & M_{21}^*(1) & M_{22}^*(1) \end{bmatrix}$$

and

$$\omega = \left(\alpha_\perp' \Xi \beta_\perp\right)^{-1} = \left[(I_{r_1} - M_{11}^*(1))\right]^{-1}$$

is a full rank matrix by the assumption that the data are at most I(1). Then the generic moving-average representation by the Granger representation theorem of the form

$$\begin{bmatrix} X_t \\ F_t \\ G_t \end{bmatrix} = C \sum_{i=1}^{t} u_i + C_1(L) \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix} + A_0$$

with

$$C = \beta_\perp \left(\alpha_\perp' \Xi \beta_\perp\right)^{-1} \alpha_\perp'$$

simplifies to (22). ■

Our model contains I(1) and I(0) factors with corresponding dynamic factors innovations. From the moving-average representation (22) we can observe that the innovations in the first group have permanent effects on X_t, while the innovations in the second group have only transitory effects. The identification of structural dynamic factor innovations can be performed separately for each group of structural innovations or on both simultaneously. As is standard in SVAR analysis, we assume that structural dynamic factor innovations are linearly related to the reduced-form innovations

$$\varphi_t = \begin{bmatrix} \eta_t \\ \mu_t \end{bmatrix} = H \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{27}$$

where H is a full-rank $(r_1+r_2)\times(r_1+r_2)$ matrix. η_t are r_1 permanent structural dynamic factor innovations and μ_t are r_2 transitory structural dynamic factor innovations. It is assumed that $E\varphi_t\varphi_t' = I$ such that $H\Sigma_{u,w}H' = I$.

The moving average representation of the FECM in structural form can be obtained by inserting the two linear transformations above of reduced-form innovations to dynamic factors to the moving-average representation of the FECM given by (22).

The three most common classes of restrictions in the SVAR literature are contemporaneous restrictions, long-run restrictions and sign restrictions. The structural FECM is first illustrated with the identification of monetary policy shocks using contemporaneous restrictions as in the original proposal of the FAVAR model by Bernanke, Boivin and Elias (2005). In this way we obtain a direct comparison of the two methods and the likely importance of incorporating cointegrating information into the FAVAR. We continue with the analysis of long-run restrictions and extend the analysis of structural common stochastic trends of King et al. (1991) to the case of large nonstationary panels. Such an identification procedure has not been discussed yet in the literature. This paper thus provides the first analysis of both a FECM with contemporaneous restrictions and the long-run scheme for the identification of structural shocks in nonstationary panels.[6]

4.2 Contemporaneous restrictions - BBE identification scheme

BBE consider the issue of identifying monetary policy shocks in large panels. The essence of their approach is in the division of variables into two blocks: slow-moving variables that do not respond contemporaneously to monetary policy shocks and fast-moving variables that do. In addition, BBE treat the policy instrument variable, the federal funds rate, as one of the observed factors. They consider two estimation methods, namely Bayesian estimation and principal components analysis. In the latter approach, most frequently used in the literature and in practice, they estimate K factors from the whole panel and from the subset of slow-moving variables only (slow factors). They then rotate the factors estimated from the whole panel around the federal funds rate by means of a regression of these factors on the slow-factors and the federal funds rate. As a result of this rotation of the factors, the analysis proceeds with $K+1$ factors, namely the K rotated estimated factors and the federal funds rate imposed a as an observable factor.

Identification of monetary policy shocks is obtained in the VAR model of rotated factors assuming a recursive ordering with the federal funds rate ordered last.

$$Cov(\varphi_t \varphi_t') = PP' \tag{28}$$

where P is lower triangular. The impulse responses of the observed variables of the panel are then estimated by multiplying the impulse responses of the factors by the loadings obtained from OLS regressions of the variables on the rotated factors. Note that this scheme identifies the structural innovations from the factors VAR only and does not impose restrictions on the loadings of the factors on the observable variables.

In addition to the inclusion of the error-correction term, an important difference between the BBE model and our model is that BBE do not account for serial correlation in the idiosyncratic components of the panel, i.e. their FAVAR model contains no lags of left-hand-side variables. In our model the presence of lagged dependent variables is a consequence of the temporal dependence in the idiosyncratic components of the factor model, and it is explicitly acocunted for in the empirical applications below.

[6] Extending to identification using sign restrictions is straightforward, but beyond the scope of this paper and is thus left for future research.

4.3 Long-run restrictions

The identification of structural innovations with long-run restrictions can be obtained by imposing restrictions on the matrices Λ and ω in the moving-average representation of the FECM (22). With this we replace the long-run effects of reduced-form innovations to factors u_t

$$\Lambda\omega\sum_{i=1}^{t}u_i$$

with the long-run effects of structural innovations denoted η_t

$$\Lambda^*\omega^*\sum_{i=1}^{t}\eta_i,$$

where the matrices Λ^* and ω^* contained restrictions motivated by economic theory. A common economically motivated identification scheme of permanent shocks, originally proposed by Blanchard and Quah (1990), uses the concept of long-run money neutrality. In this respect, their identification scheme distinguishes real from nominal shocks by imposing zero long-run effects of the nominal shock on real variables. In a cointegration framework such identification approach was formalized by King et al. (1991) (see also Warne, 1993). King et al. (1991) analyzed a six-dimensional system of cointegrated real and nominal variables. By imposing a certain cointegration rank they determined the subset of innovations with permanent effects. Within this subset they restricted the number of real stochastic trends to one, and identified it by imposing zero restrictions on real variables of all other permanent shocks in the subset. The remaining permanent shock are allowed to have non-zero effect only on the subset of nominal variables in the cointegrated VAR. We extend the identification approach of King et al. (1991) to large-dimensional panels of non-stationary data.

The FECM contains r_1 stochastic trends. Consider the case where $r_1 = 2$. We have two I(1) factors and want to identify one as a real stochastic and the second as a nominal stochastic trend. Accordingly, partition the variables in X_t such that N_1 real variables are ordered first and the remaining $N_2 = N - N_1$ nominal variables are ordered last. The group of real variables contains various measures of economic activity measured in levels, e.g. indexes of industrial production that are treated as I(1). The identifiying restrictions would thus be that the nominal stochastic trend has a zero long-run effect on these variables. Among nominal variables, for example, the panel contains the levels of different price indexes, levels of nominal wages and interest rates. Such variables are grouped at the bottom of the panel. In this case the restricted loading matrix Λ^* would have the following structure:

$$\Lambda^* = \begin{bmatrix} \Lambda^*_{11} & \mathbf{0} \\ \Lambda^*_{21} & \Lambda^*_{22} \end{bmatrix}$$

where Λ^*_{11} is $N_1 \times 1$ and Λ^*_{21} and Λ^*_{22} are $N_2 \times 1$. More generally, if the objective were to identify only the real stochastic trends with $r_1 > 2$, the dimension of Λ^*_{22} would be $N_2 \times (r_1 - 1)$. Λ^* can be identified in the following way. First, the real stochastic trend is allowed to load on all observable variables. This implies that Λ^*_{11} and Λ^*_{21} can be identified as loadings to the first factor - F^r_t - extracted from the whole dataset. Second, we can estimate the residuals from a projection of X_t on F^r_t. Denote these as ε^r_t. Then Λ^*_{22} is identified as loadings to the $(r_1 - 1)$ factors - denoted F^n_t - extracted from the lower N_2-dimensional block of ε^r_t.

Note that block diagonality of Λ^* alone does not ensure that nominal shocks do not load to real variables, but we also need (block) diagonality of ω^*. Note that it is the product $\Lambda^*\omega^*$ that determines the overall long-run effects, implying that zero long-run effect restriction requires $\Lambda^*\omega^*$ to be lower bloack diagonal, which is achieved by imposing lower (block) diagonality of ω^* in addition to lower (block) diagonality of Λ^*.

The matrix ω^* can be obtained from the estimates of the VAR model (23). We have seen above that the matrix ω can be estimated using

$$\widehat{\omega} = \left[\left(I_{r_1} - \widehat{M}^*_{11}(1)\right)\right]^{-1}.$$

Subsequently, we can identify ω^* from the long-run covariance matrix

$$\omega E(u^F_t u^{F\prime}_t)\omega' = \omega^* E(\eta_t \eta_t')\omega^{*\prime} = \omega^*\omega^{*\prime} \tag{29}$$

where $\eta_t = [\eta^{r\prime}_t, \eta^{n\prime}_t]'$ are the structural innovations and ω^* is lower block diagonal.

5 Simulation experiments

In this section we consider two simulation experiments. With the first experiment we address two questions related to the finite sample properties of the FECM estimators. We investigate whether the principal component based estimator efficiently estimates the space spanned by both the I(1) and I(0) factors. The second issue is concerned with retreiving the impulse responses to innovations to dynamic factors conditional on sample size.

With the second simulation experiment we analyze the effects of omitting the error-correction term on impulse response analysis. The data generating process for the second experiment is empirically motivated by the analysis of real stochastic trends in Section 6.2 below. The estimated responses to a permanent real shock reveal some significant differences between the FECM and the FAVAR. Given that the two models are set up such that the only difference between the two is the presence of the error-correction term, the simulation evidence presented in this section also facilitates the discussion of the empirically observed differences.

5.1 Finite sample properites of the FECM

The exact theoretical structure of (14) is rather specific. Given that the factors estimated by principal components are only a rotation of the true factors, fitting a VAR to them will not retrieve the theoretical structure given by (14) directly. This is however unnecessary, and with the simulation experiment we address two questions which enable us to attack the issue of consistency indirectly but completely. The first is how precisely PCA retrieves the space spanned by the factors in finite samples. Bai (2004) provides simulation evidence for the case with I(1) factors only and shows that the method works well also for relatively small panels. Our setting explicitly allows for both I(1) and I(0) factors and verifies the Bai simulation results in this more general scenario. Second, we test whether the impulse responses obtained from the VAR based on the estimated factors correspond to the true impulse responses obtained with the true model (14) and (7).

The design of the Monte Carlo experiment is the following. The factors are generated by a VAR such as (13) with one I(1) and one I(0) factor and two lags of each factor. The sum of the autoregressive coefficients for the I(0) factors is set to 0.7. The two factors are independent, i.e. the VAR coefficients matrices are diagonal and u_t and w_t are independent $N(0,1)$ processes. F_t and c_t enter (1) contemporaneously and with one lag, i.e. $p = m = 1$. The loadings λ_{ij}, ϕ_{ij}, $j = 0, 1$, are drawn from a standard normal distribution. Finally, the idiosyncratic component is serially correlated. This is modelled by setting the order of $\gamma_i(L)$ to two and drawing the values of γ_{i1} and γ_{i2} from $N(0.4, 1)$ and $N(0.2, 1)$ respectively.[7]

The factors are estimated from the generated Xs by principal components applied to the levels of variables, imposing the true number of factors. It follows from representation of the FECM that there is one I(1) factors - F_t, and three I(0) factors - ΔF_t, c_t and c_{t-1}.

To check whether the principal components retrieve the space spanned by the factors we follow Bai (2004) and estimate the following projection

$$\begin{bmatrix} F_t^0 \\ c_t^0 \end{bmatrix} = \delta \begin{bmatrix} \hat{F}_t \\ \hat{c}_t \end{bmatrix} + v_t$$

where F_t^0, c_t^0 denote true factors and $\hat{F}_t, \hat{c}_t$ the estimated factors. We then rotate the estimated factors towards the true factors by

$$\begin{bmatrix} \tilde{F}_t \\ \tilde{c}_t \end{bmatrix} = \hat{\delta} \begin{bmatrix} \hat{F}_t \\ \hat{c}_t \end{bmatrix}.$$

The correlation between $\tilde{F}_t$ and F_t^0, and $\tilde{c}_t$ and c_t^0 indicates how precisely PCA estimates the space spanned by the factors.

Using $\tilde{F}_t$ and $\tilde{c}_t$ we then fit a VAR of order two and estimate the parameters of the

[7] We conducted also robustness checks by varying the persistence in the idiosyncratic components. Results, available from the authors upon request, exhibit high degree of robustness.

FECM given by (7). The estimated VAR is then used to obtain the impulse responses of rotated factors to unit shocks to $\tilde{F}_t$. The resulting responses, combined with the estimated parameters of the FECM, yield the impulse responses of the Xs.

Table 1: Correlation between true and estimated factors

(1)	(2)	(3)	(4)	(5)	(6)
		Correlation between			
		$\tilde{F}_t$ and $\hat{F}_t$	$\tilde{c}_t$ and $\hat{c}_t$	$\tilde{F}_t$ and $\hat{F}_t$	$\tilde{c}_t$ and $\hat{c}_t$
T	N	I(1) variables		I(1) and I(0) variables	
30	50	0.989	0.964	0.995	0.986
50	50	0.995	0.975	0.993	0.989
50	100	0.998	0.989	0.995	0.990
50	250	0.999	0.997	0.997	0.997
50	500	0.999	0.998	0.999	0.998
100	250	0.999	0.998	0.999	0.997
100	500	1.000	0.998	1.000	0.999
100	1000	1.000	0.999	1.000	0.999
250	500	1.000	0.999	1.000	0.999
250	1000	1.000	0.999	1.000	0.999
500	100	1.000	0.993	0.999	0.994
500	250	1.000	0.997	1.000	0.998
500	500	1.000	0.999	1.000	0.999

Notes: Panel with only I(1) data in columns 3 and 4. Panel with I(1) and I(0) data in columns 5 and 6.

The impulse responses are computed for 100 periods. The VAR for the factors is estimated with the unit root imposed in the equation for $\tilde{F}_t$.[8] In order to mimic the practice in the empirical example, we do not impose the mutual independence of the (dynamic) factors.

The experiment consists of 1000 replications. Within each iteration we generate a new set of parameters and iterate 100 times on random draws of the error processes u_t, w_t and v_{it} to get the distribution of impulse responses. The confidence intervals of the impulse responses are averaged over the 1000 replications and compared to true impulse responses.

The results of the simulation experiment are presented in Tables 1 - 3 for different combinations of T and N. Table 1 reports the correlation coefficients between the true and the estimated and rotated factors. As we can see, principal components capture the space spanned by the factors quite successfully, even at moderate sample sizes. The correlations increase with both T and N.

Table 2 reports measures of coherence between true and estimated impulse responses for the two factors. In particular, columns (3) and (4) contain the share of periods the true impulse responses of both factors, either to a shock to the I(1) factors (upper panel) or a shock to the I(0) factors (lower panel), are outside the simulated 95% confidence intervals. The results show that virtually no true impulse response is outside the confidence interval of the responses to the shock to I(1) factors. The shares of responses outside the confidence interval to a shock to the I(0) factor do not exceed the theoretical 5% level. Columns (5) - (8) contain the differences between true impulse responses and the responses averaged

[8] The key results are unaltered if the unit root is not imposed in estimation. The only difference is to be found in lower efficiency (as reflected in the width of the confidence intervals). Results available from the authors upon request.

across the Monte Carlo replications, which gives a measure of the bias in finite samples.[9] Similar observations apply both to responses to a shock to the I(1) factor (upper panel), and to a shock to the I(0) factor (lower panel). We can observe that the impulse responses converge to the true responses quite fast with both T and N. As expected, also the width of the confidence intervals generally decreases with both N and T (while holding the other constant).

Table 2: Impulse responses of factors

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		% of true IRs outside 95% CI		True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	F_t	c_t	3	12	24	100	3	12	24	100
					Responses of F_t to shock to F_t						
30	50	0.0	0.0	-0.13	-0.13	-0.13	-0.13	1.46	1.54	1.56	1.54
50	50	0.0	0.0	-0.06	-0.06	-0.06	-0.06	1.13	1.14	1.14	1.14
50	100	0.0	0.0	-0.04	-0.03	-0.03	-0.03	1.02	1.03	1.03	1.03
50	250	0.0	0.0	-0.03	-0.03	-0.03	-0.03	1.05	1.05	1.05	1.05
50	500	0.0	0.0	-0.03	-0.02	-0.02	-0.02	1.01	1.02	1.02	1.02
100	250	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.93	0.93	0.93	0.93
100	500	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.92	0.93	0.93	0.93
100	1000	0.0	0.0	-0.02	-0.01	-0.01	-0.01	0.93	0.94	0.94	0.94
250	500	0.0	0.0	-0.01	0.00	0.00	0.00	0.86	0.85	0.85	0.85
250	1000	0.0	0.0	-0.01	0.00	0.00	0.00	0.85	0.85	0.85	0.85
500	100	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.84	0.83	0.83	0.83
500	250	0.0	0.0	0.00	0.00	0.00	0.00	0.83	0.83	0.83	0.83
500	500	0.0	0.0	0.00	0.00	0.00	0.00	0.83	0.82	0.82	0.82
					Responses of c_t to shock to c_t						
30	50	0.0	1.0	-0.06	-0.01	0.00	0.00	0.77	0.17	0.03	0.00
50	50	0.0	1.0	-0.08	-0.01	0.00	0.00	0.68	0.14	0.02	0.00
50	100	0.0	1.0	-0.09	-0.01	0.00	0.00	0.68	0.14	0.02	0.00
50	250	0.0	1.0	-0.07	-0.01	0.00	0.00	0.65	0.12	0.01	0.00
50	500	0.0	1.0	-0.08	-0.01	0.00	0.00	0.65	0.12	0.01	0.00
100	250	0.0	1.0	-0.10	-0.01	0.00	0.00	0.62	0.10	0.01	0.00
100	500	0.0	1.0	-0.09	-0.01	0.00	0.00	0.61	0.10	0.01	0.00
100	1000	0.0	1.0	-0.10	-0.01	0.00	0.00	0.62	0.11	0.01	0.00
250	500	0.0	3.0	-0.11	-0.01	0.00	0.00	0.58	0.07	0.00	0.00
250	1000	0.0	3.0	-0.11	-0.01	0.00	0.00	0.58	0.07	0.00	0.00
500	100	0.0	5.0	-0.12	-0.01	0.00	0.00	0.57	0.06	0.00	0.00
500	250	0.0	5.0	-0.11	-0.01	0.00	0.00	0.57	0.06	0.00	0.00
500	500	0.0	5.0	-0.12	-0.01	0.00	0.00	0.57	0.06	0.00	0.00

Notes: 1000 Monte Carlo replications

As Table 2 for factors, Table 3 reports equivalent results for impulse responses of Xs. To facilitate presentation all statistics are averaged over N variables. Also for the impulse responses of Xs we observe that practically a negligible share of impulse responses deviates from the 95% confidence intervals. The largest shares reported in column 3 are below 0.5%. These results suggest that the estimation method successfully retrieves the impulse responses to shocks. Similar observations to those of factors about the convergence of the impulse responses and their distribution apply also to the impulse responses of Xs (see columns 4 - 11 in Table 3).

Motivated by the empirical applications below, we then consider one modification to the data generating process. The dataset contains both I(1) and I(0) variables and we

[9] Note that the generated factors are independent, but independence is not imposed when working with estimated factors. Because of this, the cross-equation responses of factors are not zero, but still quantitatively limited. For this reason and in order to save space, Table 2 reports only the responses of factors to own shocks. Detailed results are available upon request.

Table 3: Estimation of impulse responses of observable variables - average across Xs

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		% of true IRs outside	True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	95% CI	3	12	24	100	3	12	24	100
				Shock to F_t						
30	50	0.00	0.15	0.23	0.24	0.25	8.35	5.96	6.06	6.06
50	50	0.00	0.06	0.09	0.09	0.09	2.48	1.89	1.87	1.87
50	100	0.00	0.07	0.05	0.06	0.06	2.68	1.89	1.90	1.90
50	250	0.00	0.07	0.04	0.05	0.05	2.68	1.71	1.71	1.71
50	500	0.00	0.09	0.02	0.03	0.03	2.49	1.66	1.65	1.63
100	250	0.00	0.10	0.01	0.02	0.02	1.28	0.70	0.70	0.70
100	500	0.00	0.09	0.01	0.02	0.02	1.30	0.69	0.69	0.69
100	1000	0.00	0.10	0.01	0.02	0.02	1.26	0.70	0.70	0.70
250	500	0.05	0.11	0.01	0.01	0.01	0.66	0.26	0.26	0.26
250	1000	0.04	0.12	0.01	0.01	0.01	0.71	0.28	0.28	0.28
500	100	0.36	0.09	0.01	0.01	0.01	0.39	0.15	0.15	0.15
500	250	0.42	0.12	0.01	0.01	0.01	0.50	0.17	0.17	0.17
500	500	0.47	0.12	0.01	0.00	0.00	0.48	0.16	0.16	0.16
				Shock to c_t						
30	50	0.00	0.36	0.04	0.04	0.13	7.14	2.40	2.51	2.55
50	50	0.00	0.35	0.02	0.01	0.01	5.02	1.75	1.78	1.79
50	100	0.03	0.35	0.01	0.01	0.00	3.98	1.45	1.50	1.50
50	250	0.03	0.37	0.01	0.01	0.01	4.23	1.55	1.61	1.61
50	500	0.03	0.35	0.02	0.01	0.01	4.13	1.45	1.49	1.51
100	250	0.07	0.32	0.02	0.01	0.00	1.81	0.89	0.90	0.91
100	500	0.09	0.34	0.01	0.00	0.00	1.97	0.97	0.98	0.98
100	1000	0.13	0.34	0.02	0.01	0.01	1.74	0.89	0.90	0.90
250	500	0.52	0.35	0.01	0.00	0.00	0.87	0.56	0.56	0.57
250	1000	0.52	0.33	0.01	0.01	0.01	0.85	0.52	0.53	0.53
500	100	1.13	0.36	0.01	0.00	0.00	0.56	0.45	0.45	0.45
500	250	0.98	0.34	0.01	0.00	0.00	0.55	0.42	0.42	0.42
500	500	0.99	0.31	0.01	0.00	0.00	0.50	0.38	0.38	0.38

Notes: 1000 Monte Carlo replications. Results in the table refer to mean impulse responses across N variables. Absolute deviations between true and estimated impulse responses.

want to investigate how the presence of I(0) variables affects the finite sample properties of the estimated factors. The setting of the experiment can be easily adapted by restricting some of the loadings of F_t to zero.

The dataset used in the empirical application contains 120 variables, 43 of which are treated as I(0). To replicate this feature we restrict roughly 36% of the loadings of F_t to zero in each sample setup. The factors are extracted from generated data using PCA without imposing the zero restrictions on the loadings.

Simulation results, presented in columns 5 and 6 of Table 1, reveal that also in the presence of I(0) variables in the panel already at moderate sample sizes PCA sucessfully retrives the space spanned by dynamic factors.

Overall, the simulation experiments indicate that principal component based estimators (with a mixture of I(1) and I(0) factors) can recover very well the factor space. Moreover, using the estimated factors in the factor VAR replicates accurately the true factor responses. Finally, inserting the estimated factor responses in the FECM, in combination with the estimated FECM parameters, delivers estimated structural impulse responses very close to the true ones.

5.2 Effects of the error-correction term

We now explore the determinants of the effects of omitting the error-correction term by means of a second simulation experiment, focusing on the role of the strength of error correction and of the sample size, along both the time series and cross section dimensions.

In the design of the data-generating process we draw from the empirical analysis of real stochastic trends that is presented in detail in the next section. The experiment is designed as follows. We estimate model (25) for the subset of I(1) variables in the panel and use the estimated parameters as DGP. The only exception are the loading coefficients of the cointegration relations, α. These are are drawn from a uniform distribution around mean values as specified below, in order to assess the effects of a different error correction strength. The idiosyncratic components of the data are treated as serially independent and bootstrapped from empirical residuals. The data are driven by factors simulated with the parameters from the estimated factors VAR, combined with bootstrapped factor VAR residuals.

Identification of the real trend requires a division between real and nominal variables in the panel. Our panel contains 55% of real variables and 45% of nominal variables. This relative share is also preserved in the artificially generated data, i.e. out of N generated variables, 55% have parameters that are randomly drawn from the parameters pertaining to real variables. The rest are randomly drawn from the parameters of the subset of nominal variables.

The results of the Monte Carlo experiment are presented in Table 4. We consider five different parameter configurations. The basic sample setup is with $T = 500$ and $N = 100$, which corresponds to the dataset from which the parameters used in the DGP are estimated. The results of the previous simulation experiment suggest, however, that we could expect reliable estimates also for other, smaller, sample sizes. The basic mean value of the error-correction coefficient α is set to -0.50. We consider four deviations from this basic parameter setup. The first two are variations in the strength of error correction, with mean α set to -0.25 and -0.75 respectively. The remaining two modifications alter the sample size. First, we halve the time series dimension to 250, and second we halve the cross-section dimension to 50. For each parameter set we take 100 random draws of the parameter set and factor process. Within each of these random draws the confidence intervals of the impulse responses are estimated through 100 bootstrap replications. The confidence intervals are used to measure the differences between the estimated impulse responses computed with the FAVAR model and those with the FECM.

The simulation results confirm our priors about the effect of the strength of error-correction. Relative to the benchmark parameter specification (columns 1 and 2), weaker error correction (columns 3 and 4) corresponds to a smaller occurrence of significant differences in the estimated impulse responses. Stronger error correction, with mean α equal to -0.75 (columns 5 and 6), conversely, leads to consistently higher occurrence of significant

Table 4: Importance of the error-correction term - results of the second Monte Carlo experiment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	-0.50		-0.25		-0.75		-0.50		-0.50	
T	500		500		500		250		500	
N	100		100		100		100		50	
Percentage of FAVAR responses outside the FECM confidence intervals										
Confidence interval coverage (%)										
Horizon	67	90	67	90	67	90	67	90	67	90
3	28.5	13.2	19.21	9.0	39.2	22.0	22.6	10.0	14.6	7.0
6	44.6	24.3	30.68	16.6	48.7	29.2	34.4	17.9	21.8	12.8
12	47.0	26.4	38.06	21.7	47.3	29.0	42.8	24.7	23.5	14.3
18	48.8	29.3	43.42	25.1	49.8	32.3	43.7	25.1	24.9	14.8
24	51.3	31.3	45.99	26.9	49.5	30.2	43.0	24.4	25.4	14.9
36	47.6	28.2	45.93	25.7	46.7	26.4	40.5	22.2	21.7	11.1
48	43.5	23.9	40.8	21.0	41.5	21.7	39.0	21.0	18.5	8.6
60	44.6	22.7	38.13	19.1	40.6	21.4	41.6	21.3	18.2	8.2
any	85.9	61.6	76.13	51.9	86.2	66.1	79.7	57.2	41.2	28.7
Average % of periods IRs outside confidence interval										
	50.4	36.4	47.9	35.6	49.1	33.8	44.9	31.3	47.9	35.0
Average partial R^2 of error-correction term (%)										
Ouside CI	3.6	3.5	2.2	2.2	4.3	4.4	8.1	8.5	3.3	3.0
Inside CI	2.2	2.9	1.7	1.7	4.5	3.7	6.2	6.6	2.2	2.8

differences at any horizon. With the strength of the error-correction increases also the average number of periods that FAVAR impulse responses remain outside the confidence interval of the FECM impulse responses.

The effect of a smaller time series dimension of the panel is not uniform across the time elapsed after the shocks. Within the first 12 periods, the differences are less frequent. At longer horizons, however, the frequency increases. The persistence in significant differences decreases slightly relative to the benchmark.

The effect of the cross-section dimension is straightforward. With fewer series in the panel, obtaining statistically different impulse responses between the FAVAR and the FECM becomes less probable. However, the persistence of those that are significantly different increases.[10]

Overall, this second experiment confirms the relevance of the inclusion of error correction terms in FAVAR models, suggesting that their omission can have sizeable effects, also in rather small panels.

6 Empirical applications

In this section we consider two empirical applications. In both we focus on the empirical importance of the error-correction mechanism for the analysis of structural shocks. The first application is about identifying monetary policy shocks using contemporaneous restrictions, where we compare the FECM to the FAVAR model of Bernanke et al. (2004). The second application uses the long-run restriction scheme to identify a real common

[10] The FECM in the simulation experiment contains 6 endogenous lags (uniform across equations), while the factors enter only contemporaneously. We repeated the same experiment also with one and three of both endogenous lags and lags of factors. The results, available upon request, are robust and fully in line with those presented in Table 4.

stochastic trend or a stochastic productivity trend.

We use the dataset of Bernanke et al. (2005). It contains 120 variables for the US, spanning over the period 1959 - 2003. 77 variables are by the authors treated as I(1). The dataset therefore contains both the I(1) and I(0) variables, which we model in the following way. Denote by X_{it}^1 the I(1) variables and by X_{it}^2 the I(0) variables. Naturally, the issue of cointegration applies only to X_{it}^1. As a consequence, the I(1) factors load only to X_{it}^1 and not to X_{it}^2. In other words, the fact that X_{it}^2 are assumed to be I(0) implies $\Lambda_i^2 = 0$, which is a restriction that we take into account in model estimation. Our empirical FECM is then

$$\Delta X_{it}^1 = \alpha_i (X_{it-1}^1 - \Lambda_i F_{t-1}) + \Lambda_i^1 \Delta F_t + \Phi_i^1 G_t + v_{it}^1 \tag{30}$$

$$X_{it}^2 = \Phi_i^2 G_t + v_{it}^2 \tag{31}$$

The model for the I(1) variables in (30) is the FECM, while the model for the I(0) variables in (31) is a standard FAVAR. As shown in Section 2.2, the space spanned by factors F_t and G_t can be consistenlty estimated using PCA on a dataset in levels containing both the I(1) and I(0) variables.

Note that (30) does not contain all the parameter restrictions of (7). It also does not include lags of factors and lags of ΔX_{it}^1 and X_{it}^2. The main reason for such a specification is the comparability with the FAVAR. In our empirical application we want to keep the specification of the FECM the same as the FAVAR of Bernanke et al. (2005) with only one exception: the error-correction term. This will allow us to evaluate the pure partial effect of error-correction mechanism on impulse response analysis. However, we also present below the results with lags of dependent variables.

The FAVAR model is in this respect as follows:

$$\Delta X_{it}^1 = \Lambda_i^1 \Delta F_t + \Phi_i^1 G_t + v_{it}^1 \tag{32}$$

$$X_{it}^2 = \Lambda_i^2 \Delta F_t + \Phi_i^2 G_t + v_{it}^2 \tag{33}$$

This is essentially the FAVAR specification of Bernanke et al. (2005). (32) differs from (30) in that it does not include the error-correction term. (33) differs from (31) by not taking into account the restriction $\Lambda_i^2 = 0$.

To provide *prima facie* evidence of the importance of the error-correction terms in (30) we tested their significance with a standard t-test equation by equation. At the 5% significance level, 63 out 77 equations have statistically significant α_i. The average partial R^2 of these terms is 2.8%, while the maximum reaches 23.4%. These figures confirm the importance of including the error-correction term in modelling variables that are originally I(1), but are modelled in differences in FAVAR applications. The average size of the partial R^2 implies a limited partial contribution of the error-correction term to the goodness of fit of the estimated equations. However, the empirical example and the Monte Carlo experiment below show that even in such circumstances omitting the error-correction terms could lead to significant distortions in estimated impulse responses.

The space spanned by F_t and G_t is estimated by the principal components on the data in levels (Bai, 2004). Our simulations reported in Section 2 give us confidence that this space is estimated consistently. Our assumption of cointegration between X_{it} and F_t is valid if the ε_{it} series is stationary. The panel unit root test (Bai and Ng, 2004) applied to our dataset rejects the null of no panel cointegration between X_{it} and F_t. In addition, the augmented Dickey-Fuller tests on individual ε_{it} largely reject the null, which leads to conclude that the method of Bai (2004) is appropriate in our setting.[11] As a robustness check we provide below also the results with factors extracted from I(0) data as in Bai and Ng (2004).

A final note is appropriate concerning the estimation of the FAVAR. In the present application, which serves to illustrate the method, we do not consider the potential dynamic singularity in the variance-covariance matrix of stationary factors G_t. A more general treatment is at present beyond the scope of this paper.

6.1 Monetary policy shocks

As described in section 5.2, the identification of monetary policy shocks is undertaken using the approach of Bernanke et al. (2005) with only one modification that makes the results obtained with the FECM directly comparable to those of the FAVAR. The difference is at the stage of factor estimation. Namely, in order to capture cointegration as in Bai (2004) we estimate the factors from the data in levels, while Bernanke et al. (2005) estimate the factors from data transformed (if necessary) to I(0).[12] This gives us the estimates of the space spanned by r_1 I(1) factors and $r - r_1$ stationary factors. As in Bernanke et al. (2005), the federal funds rate is treated as one observable factor and the estimated factors are rotated accordingly. Because their method entails identifying the monetary policy shocks from a stationary factor VAR, the first r_1 nonstationary factors are differenced. Identification of monetary policy shocks is then obtained from a VAR of stationary factors.

Bai(2004) information criteria indicate $r_1 = 2$. In the choice of the total number of estimated factors r we follow Bernanke et al. (2005) and set it to 3. However, as in their case, the main findings are robust to working with more factors. Including the federal funds rate, the total number of factors is 4.

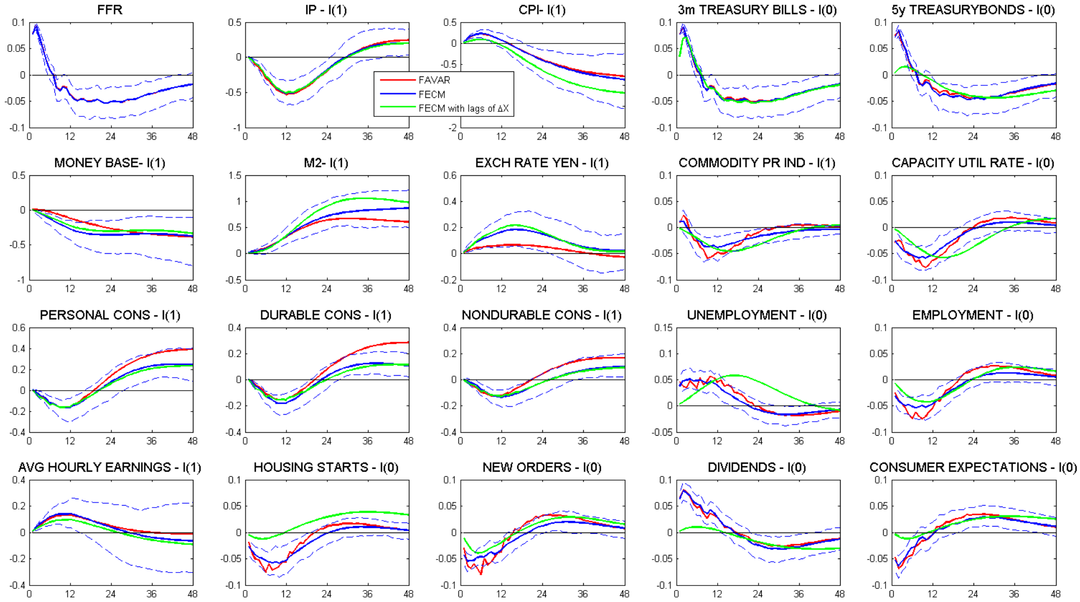
The basic results are presented in Figure 1. It contains the impulse responses for the same set of variables as in Bernanke et al. (2005) obtained from the conventional FAVAR model and the FECM model. They differ in the presence of the error-correction

[11] Results available from the authors upon request.

[12] Both approaches deliver similar estimates of monetary policy shocks. However, because the factors are estimated on datasets of different order of integration, they are not numerically identical.

term for the variables that are treated as I(1) in levels. Some variables are assumed to be I(0). These are the interest rates, the capacity utilization rate, unemployment rate, employment, housing starts, new orders and consumer expectations. For these variables the FAVAR and the FECM also differ. Consistent with (31) the FECM for I(0) variables excludes the I(1) factors. In the figure we additionally plot impulse responses obtained with a more general FECM specification in which 6 lags of ΔX_{it} are added to the model equations.

Figure 1: Impulse responses to monetary policy shock - FAVAR Vs FECM with factors extracted from levels

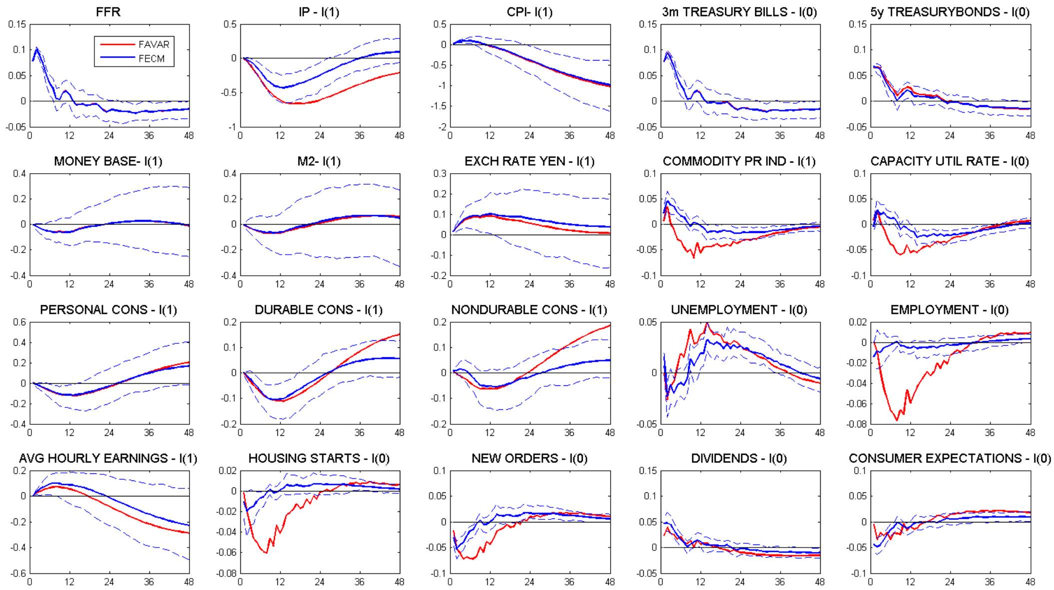


What we observe is coherence in terms of the basic shape of the impulse responses between the models. Quantitatively, however, the responses may differ significantly due to the error-correction terms. The responses of the industrial production, the CPI and wages are very similar. Quite significant differences are observed for money and the yen-dollar exchange rate. The same is true for measures of consumption. It is worth stressing that these differences are observed conditional upon a shock that accounts for only a limited share of variance. Omission of the error-correction terms in the FAVAR model can thus have an important impact on the empirical results. As we shall see below, in the analysis of real stochastic trends the differences become even more pronounced in the case of a shock that is a considerably more important source of stochastic variation in the panel.

The impulse responses of I(0) variables are very similar across models. This means that imposing the restriction that the differences of I(1) factors do not load to I(0) variables has only a limited quantitative impact, which is consistent with the FECM specification of the model. In the FECM the restriction is evident. The FAVAR that makes no distinction in the structure of the loadings of factors to I(1) and I(0) variables such a restriction cannot be directly determined.

Including endogenous lags to the FECM (green lines in Figure 1) confirms our basic findings that the omission of the error-correction term is the main source of differences in the impulse responses between the FAVAR and the FECM model.

Figure 2: Impulse responses to monetary policy shock - FAVAR Vs FECM with factors extracted from differences



As mentioned above, we also provide a robustness check of these results by estimating the factors from stationary data. In this case the identified monetary policy shocks are numerically identical to those in Bernanke et al. (2005). The results are presented in Figure 2 where the I(1) factors are estimated by cumulating the first r_1 factors estimated from I(0) panel.

The results concerning the effect of omitting the error-correction term show that the findings are relatively robust to the method of factor extraction. The main differences are that the impulse responses for monetary aggregates and the exchange rate now show a larger degree of similarity. The responses of industrial production, however, are now significantly different. These results confirm the quantitative importance of the error-correction term even if the conditioning shock is of limited importance for the overall variability in the panel.

6.2 Stochastic productivity trend

In this section we provide the results of the stochastic trends analysis as described in Section 4.2. The impulse responses to an identified permanent real shock are presented in Figure 3. The top left panel contains the responses of the real permanent trend (factor), the remaining variables are as above. Both the FAVAR model and the FECM contain six endogenous lags,[13] the only difference between the two models is the omission of the error-

[13] Robustness has been checked with respect to alternative specifications of the lag structure, namely combination of one and three endogenous lags and lags of factors. Specifications with more that three

correction term in the FAVAR. As above, for the I(0) variables the distinction between the models is the restriction that only stationary factors load to them.

The impulse responses are broadly in line with economic theory and comparable to the responses of key US macroeconomic variables to the productivity shock as reported in the DSGE model of Smets and Wouters (2007). Along the adjustment path the real factors exhibits a hump-shaped response and after three years levels off to the new higher steady state. Similar in shape are the positive responses of industrial production and measures of real private consumption. As expected, prices decrease. This effect is considerably larger in the FECM. The feature is exhibited also for other prices in the panel, but the corresponding impulse responses are not presented in Figure 3. The responses of interest rates have to opposite sign than those reported by Smets and Wouters (2007). In our case, the interest rates gradually increase. While the short rate returns to equilibrium, the effect on the 5-year return is positive, which implies a steeper yield curve. This negative price effect appears to dominate the increased demand effect on money. The responses of money are negative and again considerably more so for the FECM. Consistently with higher interest rates the dollar appreciates and more strongly so in the FECM. Consistently with the negative responses of hours worked in Smets and Wouters (2007) also in our case employment decreases slightly along the adjustment path and returns to equilibrium. Slack in the labor market correspondingly implies also a negative deviation in the average wage rate along the adjustment path. Also in this case the FECM yields a considerably stronger effect than the FAVAR.

Table 5: Percentage of FAVAR responses outside the FECM confidence intervals

Variables		**Horizon**									
	CI coverage	3	6	12	24	36	48	60	72	84	96
All	67	1.3	10.4	32.5	55.8	63.6	57.1	48.1	41.6	39.0	36.4
	90	0.0	2.6	14.3	35.1	40.3	35.1	33.8	26.0	20.8	19.5
Output	67	0.0	0.0	5.6	22.2	33.3	27.8	22.2	16.7	11.1	11.1
	90	0.0	0.0	0.0	5.6	5.6	5.6	5.6	5.6	5.6	5.6
Employment	67	0.0	0.0	29.4	58.8	70.6	58.8	35.3	23.5	23.5	17.6
	90	0.0	0.0	0.0	29.4	41.2	17.6	17.6	11.8	5.9	5.9
Consumption	67	0.0	0.0	0.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
	90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Orders	67	0.0	0.0	0.0	100.0	100.0	100.0	50.0	0.0	0.0	0.0
	90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Exchange rates	67	0.0	0.0	50.0	75.0	75.0	50.0	25.0	25.0	25.0	25.0
	90	0.0	0.0	25.0	50.0	50.0	25.0	25.0	25.0	25.0	25.0
Stock prices	67	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0
	90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Money	67	0.0	33.3	55.6	77.8	77.8	88.9	88.9	88.9	77.8	77.8
	90	0.0	0.0	33.3	66.7	66.7	77.8	77.8	77.8	77.8	66.7
Prices	67	50.0	50.0	50.0	100.0	100.0	100.0	100.0	100.0	50.0	50.0
	90	0.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
Wages	67	0.0	20.0	60.0	100.0	100.0	93.3	93.3	93.3	80.0	66.7
	90	0.0	6.7	26.7	80.0	93.3	93.3	86.7	53.3	33.3	26.7

More detailed statistics across different categories of variables and overall are given in

lags of factors were not considered in order to avoid overfitting. Becasue the model contains four factors, including up to three lags in addition to contemporaneous terms implies sixteen terms with factors in each equation. Results obtained with alternative lag structures of the FECM and teh FAVAR show a great degree of similarity to the results presented in Table 5 and Figure 3. Results are available upon request.

Figure 3: Impulse responses to real stochastic trend - FAVAR Vs FECM

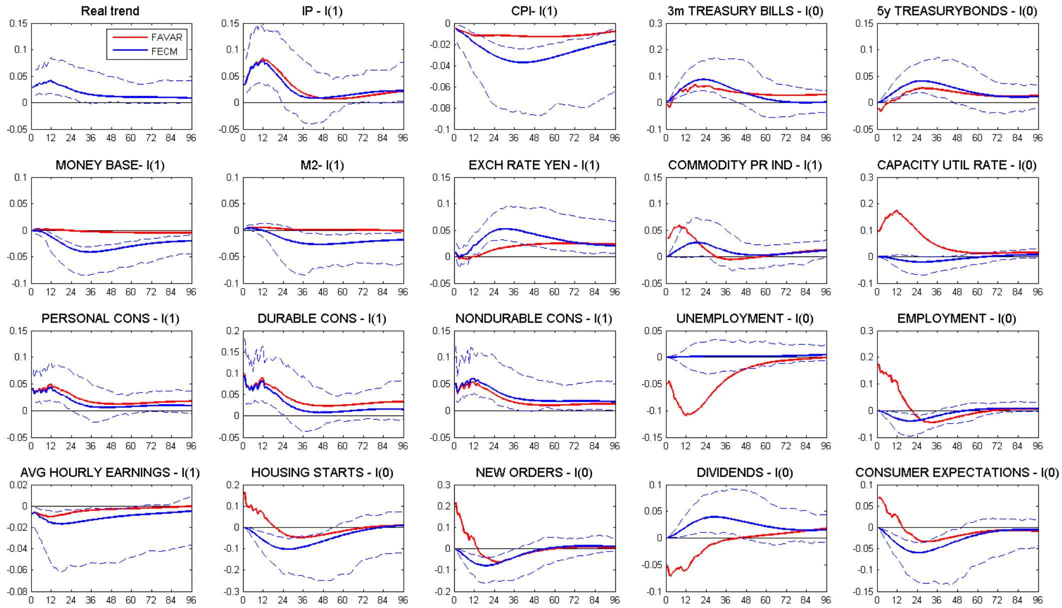


Table 5. It reports the percentage of variables (out of 77 I(1) variables in the panel) for which the impulse response obtained with the FAVAR model lie outside the confidence interval of the FECM impulse responses at different horizons. Taking into account all 77 I(1) variables, we observe that within the first 6 months after the shock only a limited number of impulse responses differ significantly. At the 12-month horizon roughly a third of impulse responses differ at 67% confidence and 14% at 90%. For the three-year horizon, these shares increase to 64% and 40% respectively. With further increases of the time horizon, the shares of statistically significant responses decrease, which is a logical consequence of increasing width of the confidence intervals.

Across categories of variables, we observe the largest shares of statistically significant differences in the impulse responses for prices, monetary aggregates, exchange rates, employment and wages. It is only for measures of output and private consumption that we see that neglecting cointegration between variables and factors has only a limited effect on the impulse responses analysis. For the remaining variables significant differences are frequent and quantitatively important.

7 Conclusions

In this paper we analyse the implications of cointegration in structural FAVAR models. Starting from a dynamic factor model for non-stationary data, we derive the factor-augmented error-correction model and its moving-average representation. Structural analysis is based on two sets of identification restrictions. In the first, we adapt the Bernanke et al. (2005) identification of monetary policy shocks to the FECM framework. While qualitatively similar, in comparison to Bernanke et al. (2005), the results can be quantitatively quite different. The differences are even more pronounced under the second identification scheme, namely the analysis of stochastic trends with long-run restrictions. Here the paper provides the first analysis of this class of restrictions in the context of cointegrated panels. In an empirical example we identify the real stochastic trend and show that accounting for cointegration has important effects on the impulse responses to this trend.

These findings are confirmed also by means of simulations experiments. Simulation results show that the differences between the impulse response functions obtained by the FECM and the FAVAR are more pronounced the higher is the strength of the error-correction and the higher is the cross-section dimension of the panel. The effect of the time series dimension is less pronounced.

Overall, these results suggest that the FECM that exploits the information in the levels of nonstationary to explicitly model cointegration provides an empirically important alternative to classical FAVAR models in structural modelling. Its implications could be extended to other identification schemes such as sign restrictions. A detailed analysis of these is beyond the scope of this paper.

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