Effects of Fiscal Policy in the DSGE-VAR Framework:  
The Case of the Czech Republic

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23 March 2016

<Preliminary and incomplete>

Abstract

In this paper we explore the potential of the DSGE-VAR modelling approach to examine the effects of fiscal policy. The combination of VAR and DSGE frameworks leads theoretically to more accurate estimates of impulse responses and consequently of fiscal multipliers. Moreover, the framework allows discussion about the differences of the effects of fiscal shocks in DSGE and VAR models and the extent of misspecification in fiscal DSGE models. The DSGE-VAR model is estimated on Czech data covering the period from 1996 to 2011 at quarterly frequency. The results suggest that misspecification of a standard fiscal DSGE model is present. Overall, the DSGE-VAR model leads to larger fiscal multipliers compared to those from the DSGE counterpart. The difference is particularly pronounced in the case of government consumption multipliers, which are about twice larger in the DSGE-VAR (0.80–0.92) as compared to the DSGE (0.42–0.47).

JEL Codes: C11, E62, F41, H30.

Keywords: DSGE-VAR model, fiscal multipliers, fiscal shocks identification, model misspecification

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This work was supported by Czech National Bank Research Project No. B3/14. We thank Róbert Ambriško and Miroslav Plašil for helpful comments. The paper benefited from comments at the seminar at the Czech National Bank. All errors and omissions are ours. The views expressed are those of the authors and do not necessarily reflect the views of the Czech National Bank.
1. Introduction

Assessment of the effects of fiscal measures on the economy represents one of the topical issues. Such effects are quantified in terms of fiscal multipliers, which characterise the reaction of output to changes in selected fiscal instruments on the revenue and expenditure side. The number of studies, which estimate the values of fiscal multipliers for different countries, time periods, and using various methods, is rapidly increasing. The variety in the estimated size of fiscal multipliers even became a subject of research by means of meta-regression analysis – a systematic quantitative literature review – with an objective to identify some regularity in the large amount of multipliers produced. Based on the review of 104 primary studies, Gechert (2015) reveal that the size of fiscal multipliers depends, inter alia, on the method selected (time series approaches or vector autoregressions (VARs) result in higher values of multipliers compared to structural macroeconomic models), on the fiscal instrument (the highest multipliers typically associated with government investment), and on the share of the constrained households who consume all their disposable income (higher share of such households strengthens the effects of fiscal policy).

Three alternative frameworks have been particularly popular in the literature investigating the effects of fiscal policy: (i) structural vector autoregression (SVAR) models (e.g. Blanchard and Perotti, 2002); (ii) narrative approach (Romer and Romer, 2010); (iii) dynamic stochastic general equilibrium (DSGE) models (e.g. Gali et al., 2007). These frameworks differ in the way the fiscal shocks – that is the unanticipated policy changes – are treated. In SVAR models, fiscal shocks are identified within an unrestricted system; the narrative approach identifies fiscal shocks directly, by selecting those fiscal policy changes which are deemed to be exogenous; DSGE models identify fiscal shocks taking into account micro-level agents’ behaviour implying cross-coefficient restrictions.

Standard SVAR techniques are often found to lack robustness due to relatively short and noisy fiscal data. Hence, Bayesian techniques become attractive as they allow incorporating additional information into the estimation procedure by imposing priors on the model parameters. The choice of priors, though, is of critical importance. A common approach of selecting VAR priors based on time series or statistical criteria has been criticised for a lack of economic interpretation. For example, a popular Minnesota prior takes an assumption that the series simply follows a random walk. Such type of prior ignores any potential interdependencies between the endogenous variables (Del Negro and Schorfheide, 2004).

A natural way how to make priors economically consistent is to use a macroeconomic model to formulate them. Although this idea is present already in Ingram and Whiteman (1994) who consider the RBC model, it took another decade to make this approach operational. In their influential study, Del Negro and Schorfheide (2004) introduce the methodology of the DSGE-VAR approach. They use a standard small-scale New-Keynesian DSGE model to generate a
prior for a vector autoregression to examine the effects of monetary policy. Since then the methodology has been employed several times mainly for forecasting.1

Another issue that could be at least partially resolved by employing the DSGE-VAR framework is that of shock identification. The identification of fiscal shocks is inherently problematic and no agreement exists on an appropriate identification scheme. The micro-founded DSGE model provides through the cross-coefficient restrictions necessary theory that help to link estimated residuals to structural shocks and thus to identify fiscal shocks within the VAR part of the model. Importantly, as discussed in details in Del Negro et al. (2007) such bridge between unrestricted VAR and VAR implied by the DSGE model allows discussing misspecification of the structural model.2

The contribution of this paper is threefold. First, to our best knowledge the paper represents a first attempt to apply DSGE-VAR approach to the analysis of fiscal policy. So far, the focus has been solely on monetary DSGE-VARs. Second, the DSGE-VAR framework is used for the examination of the misspecification of a standard fiscal DSGE model and thus can enrich the discussion on the set up of fiscal DSGE models in general. Finally, the comparison of fiscal multipliers implied by the DSGE model and DSGE-VAR model can shed some light on the discussion about differences in multipliers in DSGE and VAR modelling frameworks.

We estimate the model on Czech quarterly data covering the period 1996–2011. The end of the sample is chosen to avoid the zero lower bound (ZLB) period, in which fiscal multipliers presumably changed significantly. The short-time series available strengthen the importance of the Bayesian estimation and an appropriate formulation of priors. The fiscal DSGE model employed is a medium-scale model with all parts which are currently standard in structural modelling of fiscal policy. The size of the model along with the short-time series would be a problem for forecasting. However, our focus is on policy analysis and fiscal multipliers. Therefore, detailed modelling of various channels is advantage even with the low number of observations used for the estimation.

In our analysis we distinguish several fiscal measures on the expenditure side (government consumption shock, government investment shock, other social benefits shock) and on the revenue side (consumption tax shock, wage tax shock). The results show that impulse responses based on DSGE and DSGE-VAR models exhibit a number of differences. This suggests the presence of misspecification bias in the DSGE part considered alone.

Cumulative fiscal multipliers implied by the DSGE part attain their largest values for government consumption (0.42–0.47), followed by government investment (0.18–0.19) and other social benefits (-0.10– -0.29). The combination of DSGE and VAR approaches leads to

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1 Applications of the DSGE-VAR approach focus mainly on forecasting and often suggest superior performance with respect to standard benchmarks. Lees et al. (2011) apply the DSGE-VAR methodology to estimate the 5-variable system for New Zealand, Gupta and Steinbach (2013) develop the DSGE-VAR model of South Africa, and Langcake and Robinson (2013) develop a multi-sector DSGE-VAR model for Australia comprising 10 endogenous variables.

2 Recently the misspecification analysis introduced in Del Negro et al. (2007) was applied, for example, on misspecification related to expectations in Cole and Milani (2014).
about twice larger multipliers for government consumption (0.80–0.92), very similar
government investment multipliers (0.18–0.20), and somewhat stronger other social benefits
multipliers on the impact (-0.15) yet dying off rapidly (-0.02 since the 4th quarter).

The rest of the paper is organised as follows. After this introduction, Section 2 describes the
underlying DSGE model and Section 3 presents the VAR counterpart. Section 4 discusses the
identification of shocks, issues related to definition of fiscal multipliers are discussed in
Section 5. Section 6 summarises the data used in the DSGE and VAR parts and estimation
procedures. Section 7 presents the results of the DSGE-VAR, showing the impulse responses
and the obtained multipliers. The last section concludes. Finally, the computation of marginal
likelihood is presented in Appendix A, the list of model variables with their definitions is
provided in Appendix B and converge diagnostics of the model is discussed in Appendix C.

2. Model – The DSGE Part

In this section we provide a summary of the DSGE model from which we intend to form
priors for the DSGE-VAR estimation. More detailed description of the DSGE model can be
found in Ambriško et al. (2015) and Ambriško, Babecký, et al. (2012).

The model blocks of the structural (DSGE) model have been built along the lines of CNB’s
g3 model as put forward by Andrle et al. (2009). The model is based on a small open
economy set-up in which the foreign environment is assumed to be strictly exogenous in
econometric sense. The central bank operates in an inflation targeting regime and the interest
rate is set according to the forward-looking Taylor rule. The implementation of the production
structure of the economy mimics the main links within the system of national accounts in that
capital and labour are the sources of domestic intermediate production, which – taken together
with the imports – forms the input for the final use sectors: consumption, investment and
exports. The model is closed by an uncovered interest parity condition augmented by an
adjustment term sensitive to the net foreign asset position of the economy (so called debt-
elastic risk premium).

The modelling approach of all sectors in the economy shares identical strategy – the sector
inputs get aggregated using a constant elasticity of substitution production function, and the
pricing in all sectors builds on the sticky prices premise, as introduced by Calvo (1983):

\[
\int_0^1 X_t \left( \frac{\omega_X}{\gamma_X} \right)^{\frac{1}{\gamma_X}} \left( N_t \right)^{\frac{1}{\gamma_N}} \left( Y_t \right)^{\frac{1}{\gamma_Y}} \frac{\gamma_N - 1}{\gamma_N} \right] \frac{d z_X}{z_X},
\]

\[\log \frac{\Pi_t^X}{\Pi_{t-1}^X} = \beta \log \frac{\Pi_{t+1}^X}{\Pi_t^X} + \left( 1 - \varepsilon_X \right) \left( 1 - \beta \varepsilon_X \right) \log \left( RMC_t^X \Theta_t^X \right) + \varepsilon_t^X, \]
where $X$ stands for a sector label (i.e. intermediate production, private consumption, investment and exports sectors), $z^X$ is a sector-specific firm identifier$^3$, $\omega_x$ is the share of imported input, $N^X_t$, $Y^X_t$ represent the amounts of imported/domestic production inputs, $\eta_x > 0$ is the elasticity of substitution parameter between the domestic and foreign inputs. $\Pi^X_t$ denotes the price growth in sector $X$, which is a function of the real marginal costs $RMC^X_t$, $\varepsilon^X_t$ is a cost-push shock, and $\beta$, $\varepsilon^X$, and $\Theta^X$ are fixed parameters.

The core model behaviour is influenced by the treatment of the households sector. Following Gali et al. (2007) we assume two types of households – optimisers and “rule-of-thumb” consumers. The households with optimising behaviour tend to generate savings based on current economic conditions and their expectations. The other type of households is treated as non-Ricardian and always consumes its entire disposable income. We also adopt the recipe of Coenen et al. (2012) in that the households’ utility$^4$ is partially affected by consumption of government sector goods:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),$$

$$C_t = \left[ \left( \alpha_c \right)^{v_c-1} \left( C_t^p \right)^{v_c-1} + \left( 1 - \alpha_c \right)^{v_c} \left( G_t \right)^{v_c} \right]^{v_c},$$

where the consumption of households, $C_t$, is a CES aggregate of the output from private consumption goods producers, $C_t^p$, and government goods, $G_t$.

The division of households into two types results in a pair of budget constraints. The “rule-of-thumb” households consume the entire after-tax income from their supplied labour (augmented by the unemployment and other benefits). On top of this specification, the optimising households split their income into consumption and investment. The investment expenditures are furthermore divided into capital investment and purchases of domestic bonds. On the income side, the optimising households gather yields from previous investment, besides the after-tax labour income. Specifically, the budget constraints are as follows:

$$\text{optimisers: } \left( 1 + \tau^c_t \right) P^C_t C_t^{po} + P^I_t I_t^{po} + B_t^o = \left( 1 - \tau^c_t + \tau^UB_t \right) \int_0^T W_t(L_t) L_t^o \text{d}t + P^C_t \left( OB_t^o - T_t^o \right)$$

$$+ \left( 1 - \tau^k_t \right) P^K_t + \tau^k_t \delta P^I_t \left[ K_t^{po} + R_{t+1} B_{t+1} - \left( 1 - \tau^D_t \right) D_{t+1} \right],$$

$^3$The usual strategy is to index all sector-specific agents on a unit mass continuum so that the sector aggregates equal per capita values.

$^4$The utility function itself furthermore features the habit formation in consumption.
“rule-of-thumb”: \[
\left(1 + \tau^r_t\right)P^{\text{C}}_t C^{pr}_t = (1 - \tau^w_t + \tau^{UB}_t) \int_0^t W_t(i) \xi_t(i) di + P^{\text{C}}_t \left(\text{OB}_t - T^r_t\right),
\]
where \(\tau^r_t, \tau^w_t, \tau^K_t, \tau^D_t\) represent the tax rates (consumption, wage, capital and dividend, respectively), set by the government and \(\tau^{UB}_t\) is the unemployment benefit rate; \(C^{po}_t\) and \(C^{pr}_t\) denote private consumption of optimising and "rule-of-thumb" households respectively; \(I^{po}_t\) is optimisers’ investment in private capital \(K^{po}_t\), the depreciation rate of capital is denoted by \(\delta^p\); \(W_t\) is the wage rate and \(L_t\) is the amount of the supplied labour; Domestic bond purchases are labelled as \(B^{o}_t\), \(R_t\) is the nominal interest rate; The term \((\text{OB}_t - T^r_t)\) represents all benefits (other than unemployment benefits) free of lump-sum taxes; \(D^{o}_t\) are dividends from monopolistic firms; and \(P^{\text{V}}_t\) stands for sector specific prices.

The labour market is embedded into the model using the concept of Galí (2011) according to which unemployment is a result of workers’ market power and unemployment fluctuations arise because of the existence of nominal rigidities. We model the development of wage dynamics using a Calvo-type Phillips curve (similar to equation 2.2) in which we assume a simple formula in place of the marginal costs term:

\[
RMC_t = (u_t)^\varphi,
\]
where \(u_t\) is the unemployment rate and \(\varphi\) is the sensitivity parameter.

The fiscal authority has under its control a set of fiscal instruments (i.e. various tax rates and individual expenditure components). Following Leeper et al. (2010) the fiscal block is modelled by simple backward-looking rules. Each instrument at time \(t\), \(X_t\), is a function of the output gap, \(Y_t\), and the gap of indebtedness, \(B_t\), and cross-correlations among the instruments are also allowed for. Specifically:

\[
gap(X_t) = \text{gap}(Y_t)^{\psi_x} \text{gap}(B_t)^{\psi_x} \left(\left[u_t\right]^{\psi_x} \left[u_t\right]^{\psi_x} \ldots \left[u_t\right]^{\psi_x}\right),
\]
where the sequence of shocks \(u_t^{[1]}\) can potentially influence more instruments at one time. This way we model the gaps of government consumption, government investment, social benefits expenditures and the tax rates.

The government furthermore operates subject to its budget constraint, which states that the current debt stock (as of time \(t\)) is given by the debt stock in the previous period augmented by the associated interest payments and the actual primary surplus (–) / deficit (+), or:

\[
B_t = R_{t-1} B_{t-1} + PD_t,
\]
\[
PD_t = P^{G}_t G_t + P^{I}_t I^{s}_t + \tau^{UB}_t W_t L_t -
\]
where the primary deficit, \( PD_t \), is taken as the difference between the government expenditures and the government revenues (thus this measure can be negative); \( G_t \) and \( I_t^g \) stand for government consumption and investment respectively.

3. Model – The VAR Part

Following Del Negro and Schorfheide (2004) we consider VAR(p) model for \( n \times 1 \) vector of observed variables \( y_t \):

\[
y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t,
\]

where \( u_t \sim N(0, \Sigma_u) \) is an \( n \times 1 \) vector of error terms. Defining the \( T \times n \) matrix \( Y \equiv [y'_1, \ldots, y'_T] \) and \( T \times (1 + np) \) matrix \( X \equiv [x_1, \ldots, x_T] \) where \( x_t = [y_{t-1}', \ldots, y_{t-p}'] \) and \( \Phi \equiv [\Phi_0, \ldots, \Phi_p] \), the system (3.1) can be rewritten into the matrix form:

\[
Y = X\Phi + U,
\]

where \( U \) is a \( T \times n \) matrix with \( u'_t \) in rows.

The DSGE part of the model provides priors for the VAR in (3.2) such that if we define functions:

\[
\Phi^*(\theta) \equiv \Gamma^{x-1}_{xx}(\theta)\Gamma^{x}_{xx}(\theta)
\]

\[
\Sigma^*_u(\theta) \equiv \Gamma^{x+1}_{xx}(\theta) - \Gamma^{x+1}_{xx}(\theta)\Gamma^{x-1}_{xx}(\theta)\Gamma^{x}_{xx}(\theta)
\]

then the prior distribution of the VAR parameters \( \Phi \) and \( \Sigma_u \) is of the Inverted-Wishart – Normal form:

\[
\Sigma_u | \theta \sim IW(\lambda T \Sigma^*_u(\theta), \lambda T - k, n)
\]

\[
\Phi | \Sigma_u, \theta \sim N(\Phi^*(\theta), \Sigma_u \otimes (\lambda T \Gamma^{x*}_{xx}(\theta))^{-1})
\]

\( \Gamma^{x*}_{xx}(\theta), \Gamma^{x+1}_{xx}(\theta) \) and \( \Gamma^{x*}_{xx}(\theta) \) are population moments implied by the DSGE model: for example \( \Gamma^{x*}_{xx}(\theta) = E_{\theta}[x_t x'_t] \). They can be computed analytically from the state space representation of the DSGE model.

The posterior distribution of the VAR parameters then belong the same family of distributions. More precisely, it follows the following distributions:

\[
\Sigma_u | Y, \theta \sim IW((\lambda + 1)\Sigma_u(\theta), (\lambda + 1)T - k, n)
\]

\[
\Phi | Y, \Sigma_u, \theta \sim N(\Phi(\theta), \Sigma_u \otimes (\lambda T \Gamma^{x*}_{xx}(\theta) + XX)^{-1})
\]
where $\tilde{\Phi}(\theta)$ and $\tilde{\Sigma}_u(\theta)$ are the maximum likelihood estimates based on the data sample and sample produced by the DSGE model with the parameter vector $\theta$:

$$
\tilde{\Phi}(\theta) = \left( \lambda IT_{xX}^*(\theta) + X X^T \right)^{-1} \left( \lambda IT_{xX}^*(\theta) + X Y \right)
$$

$$
\tilde{\Sigma}_u(\theta) = \frac{1}{(\lambda + 1)^2} \left[ \left( \lambda IT_{xX}^*(\theta) + Y X \right)^{-1} \left( \lambda IT_{xX}^*(\theta) + Y X \right) \right]
$$

Parameter $\lambda$ is obtained by maximizing the marginal likelihood of the DSGE-VAR model. The computation of the marginal likelihood is described in Appendix A.

4. Identification of Shocks

Identification of shocks basically means to find a linear relationship between uncorrelated structural shocks $e_i$ and model residuals $u_i$. In VAR models, the relationship can be expressed as follows:

$$
u_i = \Sigma_v \Omega e_i,$$

where $\Sigma_v$ is an $n \times n$ lower triangular matrix obtained by the Cholesky decomposition of the estimated variance-covariance matrix $\Sigma_u$ and $\Omega$ is an arbitrary $n \times n$ orthonormal matrix. The choice of $\Omega$ is usually based on some theoretical considerations. For example, traditional recursive identification scheme that imposes no contemporaneous reaction of some variables to some shocks is obtained when $\Omega$ is equal to an identity matrix.

When looking for a theoretically grounded $\Omega$, the theory included in the micro-founded DSGE part of the model can be employed. So, Del Negro and Schorfheide (2004) suggest using orthonormal matrix $\Omega_{DSGE}(\theta)$ from the DSGE model which can be obtained from the unique LQ factorization of the matrix of impacts of shocks $A_v(\theta)$:

$$
A_v(\theta) \equiv \frac{\partial \gamma_i}{\partial \varepsilon_{DSGE}} = \Sigma_{w}^{DSGE}(\theta) \Omega_{DSGE}(\theta).
$$

The relationship between structural shocks from the DSGE model $e_i^{DSGE}$ and estimated one-step-ahead forecast errors $u_i$ is then expressed by the following formula:

$$
u_i = \Sigma_v \Omega_{DSGE}(\theta) e_i^{DSGE}.
$$

Since the DSGE model includes 30 structural shocks $e_i^{DSGE}$ and 26 observed variables, the lower triangular matrix $\Sigma_v$ is extended by the block of zeros on the right edge of the matrix in order the matrix to be of the size compatible with $\Omega_{DSGE}(\theta)$. In this way, we impose a zero
effect of the last 4 DSGE structural shocks. The effects of those shocks are not of interest. However, imposing zero effects of some shocks affects the results as the unexplained variation in observed variables is linked to lower number of shocks. The strength of the influence is related to the extent how much variation is explained by the neglected shocks in the data generating process. The quantification of the effect is beyond the scope of this paper.

There are several theoretical reasons why impulse response functions (IRFs) and consequently fiscal multipliers could be estimated more accurately using the DSGE-VAR framework. First of all, the DSGE-VAR framework relaxes cross-coefficient restrictions form the DSGE model and let data speak through the unrestricted VAR. In this way, potential misspecification included in the DSGE framework can be dealt with and DSGE-VAR model parameters should be estimated more accurately. Of course, if the underlying DSGE model describes the data-generating process perfectly, the VAR part does not add anything and parameters of the VAR corresponding to the DSGE model do not change.

Next set of reasons for the superiority of the DSGE-VAR framework is related to shock identification itself. Identification of fiscal shocks in VAR models is contentious issue and different identification schemes lead often to completely different results (see e.g. Franta, 2012). Taking over the identification scheme from theoretically sound DSGE model is, therefore, desirable.

In addition, the identification combining theory from a DSGE model with a-theoretical data-driven VAR model could help to address the issue of fiscal foresight in fiscal shock identification. Fiscal variables usually reflect shocks with a lag as the implementation of announced fiscal changes takes some time. This problem is even more profound in quarterly data comparing to yearly data. Ignoring the anticipation of fiscal changes by economic agents can lead to inconsistent IRFs (Leeper et al., 2012). DSGE model which imposes rational expectations can identify fiscal shocks more accurately because the DSGE model with rational agents expecting a fiscal shock the next period can attribute the change in variables to the shock already in the current period. The VAR could assign such change to other types of structural shocks. On the other hand, in DSGE models fiscal foresight can result in a situation when the observed stochastic process cannot be represented by a VAR structure (structural shocks are non-fundamental) – see e.g. Lippi and Reichlin (1993).
5. Multipliers

In general, fiscal multipliers are defined as a change in economic activity variable for a change in a fiscal policy instrument. Since fiscal tools affect economy with a lag, cumulative multipliers at specific horizons are usually considered. The cumulative multiplier is defined as the cumulative change in economic activity over the cumulative change in a fiscal policy instrument at a given horizon.

As an economic activity variable we employ the sum of private consumption (PC), government consumption (GC), private investment (PI) and government investment (GI). Such measure coincides with GDP for a closed economy. The list of fiscal measures considered in our analysis includes government consumption, government investment, and other social benefits (OSB).\(^5\) The change of the instrument is represented by a respective unexpected one-period unit shock.

The involved variables in the model are expressed in log-differences with zero as their steady state. Impulse response functions are defined in terms of the difference from the steady state i.e. in percentage points. To obtain multipliers the ratio of cumulative differences need to be multiplied by the average ratio of the activity measure and the respective fiscal measure. More precisely, first we compute the fiscal multiplier for components of economic activity measure at horizon \(h\). For example, for the government consumption as a component of activity variable and other social benefits as a fiscal measure, the fiscal multiplier is:

\[
F_{GC}(h) = \frac{\Delta GC}{\Delta OSB} = \frac{CIRF_{GC}(h) \ast SS_{GC}}{CIRF_{OSB}(h) \ast SS_{OSB}},
\]

where \(CIRF_i(h)\) is the cumulative impulse response at horizon \(h\) and \(SS_i\) is steady state value of the respective variable \(i\). The ratio of the two steady state values is approximated by the observed mean ratio in the data. The implicit assumption is that this ratio is not changing much over time, which is demonstrated on Figure 1 (panels A–C). The total fiscal multiplier at horizon \(h\) is then the weighted average of respective economic activity components:

\[
F(h) = w_{GC} F_{GC}(h) + w_{GI} F_{GI}(h) + w_{PI} F_{PI}(h) + w_{PC} F_{PC}(h).
\]

Again, weights are approximated by the observed respective ratios, which do not change much over time as it is demonstrated on Figure 1 (panel D).

\(^5\) Other fiscal measures, such as the unemployment benefits, consumption tax revenues and wage tax revenues could be potentially examined as they are included in the set of endogenous variables of the model. However, the computation of respective fiscal multipliers is problematic because the measures equal to a product of two model variables. For example, consumption tax revenues are product of the consumption tax rate and consumption.
Figure 1: Ratios of components of activity measure and fiscal measure (panel A-C) and weights of components of activity measure (panel D)

Panel A: Denominator – Government Consumption

Panel C: Denominator – Government Investment

Panel C: Denominator – Other Social Benefits

Panel D: Denominator – Economic Activity


6. Data and Estimation

The data are used at quarterly frequency, covering the period from 1996 to 2011. The underlying DSGE model comprises 26 endogenous variables. These variables and structural shocks are listed in Appendix B. The features of the Czech fiscal data and details on adjusting for data quality such as the use of the Kalman filtration and application of manual adjustment for one-off government transactions are discussed in Ambriško, Babecký, et al. (2012).

The DSGE part of the model is estimated using Bayesian techniques. We keep approximately half of the model parameters calibrated and for the rest of the parameters we apply the following strategy. As for the prior distributions we assume normal distribution for the majority of model parameters. Due to the zero lower bound in case of the standard deviations of the shocks we apply inverse gamma prior distributions. Posterior modes of the estimated parameters are derived numerically based on the likelihood maximisation principle and consequently we use the Metropolis-Hastings (MH) algorithm to learn about the posterior distributions. In order to ensure sufficient convergence, we run the MH procedure twice, each run consisting of 200,000 draws. Both simulations yield similar acceptance rates (over 50%).
The VAR part of the estimation is based on a standard Gibb sampler employed for the case of the inverted-Wishart – Normal prior. The posterior distributions are known in closed form and 5,000 draws are taken for posterior inference.

Convergence diagnostics of the sampler for the DSGE-VAR model is discussed in Appendix C.

7. Results

As an example, we start with the illustration how the combination of DSGE model and VAR model within the DSGE-VAR framework affects the estimated VAR parameters. Figure 2 reports posterior distributions of the coefficient at the first lag of selected variables in the equation with the interest rate (Pribor) on the left hand side. Parameter \( \lambda \) drives the tightness of the prior in the VAR part of the model around the parameter vector implied by the DSGE part of the model. So, for lower values of \( \lambda \) (left panel) one can observe that data within the unrestricted VAR change the prior implied by the DSGE model with imposed cross-coefficient restrictions. For example, the VAR part suggests higher smoothing parameter in the interest rate rule (\( pribor_{t-1} \)) or it suggests a non-zero reaction of the interest rate on the observed inflation which is ruled out by the structure of the interest rate rule in the DSGE model (\( defC_{t-1} \)). For high values of lambda (right panel) the prior implied by the DSGE model is very tight and the VAR part does not affect the prior almost at all – posterior means coincide with prior means.

**Figure 2: Posterior distributions for different values of tightness of the prior**

![Posterior distributions for different values of tightness of the prior](image)

*Note:* Posterior distributions of coefficient at first lag of the exchange rate (czkeur), the interest rate (pribor), and consumption deflator (defC) in the interest rate equation for \( \lambda = 8 \) (left panel) and \( \lambda = 100 \) (right panel).

The illustration provided by Figure 1 suggests that the choice of the value of the parameter \( \lambda \) plays an important role. The value is chosen based on the marginal likelihood measure. Figure 3 reports the evolutions of marginal likelihood over values for \( \lambda \) for different number of lags in the VAR part. First of all, the marginal likelihood suggests using four lags (\( p = 4 \)). Furthermore, the maximum marginal likelihood is obtained for \( \lambda = 28.7 \) which means rather
tight prior in the VAR part. The high value of $\lambda$ suggest that the DSGE model fits data well. On the other hand, the short time series available are reflected by the relative tightness of the prior. So, the value of the parameter is probably a combination of both.

**Figure 3:** Graphs of the marginal likelihood for DSGE-VAR($p$) model with different number of lags in the VAR part

![Graphs of the marginal likelihood](image)

7.1 Impulse responses

We present the results for both the impact matrix $A_{\theta}(\theta)$ taken over from the DSGE model completely and the combination of the VAR lower triangular matrix and rotation from the DSGE model: $\Sigma_{\theta}^{DSGE}(\theta)$. The reason is that such a distinction allows comparison of the IRFs and fiscal multipliers between the DSGE and VAR approaches. For both cases, the coefficients at lagged values of variables are used those from the DSGE-VAR model.

Rossi and Zubairy (2011) demonstrated for the US data how important it is to consider both monetary and fiscal policy shocks for an appropriate estimation of the effect of either of them. Our model includes both fiscal and monetary policy variables and thus from this point of view is suitable for both monetary and fiscal shock identification. In addition, checking the effects of monetary policy shocks (which are examined in much greater detail than fiscal shocks in the literature) could provide some justification for the estimated effects of fiscal shocks.
Figure 4 presents the IRFs of the monetary policy shock of size 1 (emp), i.e. an unexpected one-quarter one percentage point increase of the interest rate. The responses of the DSGE model (i.e. with the shock impacts completely taken over from the DSGE model) not surprisingly follow standard patterns – immediate appreciation of the exchange rate (czkeur), a fall in consumption deflator (defC) and consumption (diffC). Relaxing restrictions imposed by the theory leads to changes in the responses of the exchange rate and consumption deflator. The so-called price puzzle – a temporarily increase in prices following an unexpected increase in the interest rate – can be observed from the DSGE-VAR model. Patterns in data do not justify restrictions imposed by the DSGE model. Possible explanation is that we do not control for some variables important for the evolution of inflation and the VAR model assigns the variation to monetary policy shocks. The problem is fixed in the DSGE model by an imposed restriction, while in the VAR framework we need to add the variable we do not control for. Within the DSGE model it is an acceptable solution to impose the restriction, but the problem is that when we fix the reaction of prices on the interest rate, we have no idea which other reactions get affected (through the restriction on the reaction of prices). And there is no reason that the influence is in the right direction.

**Figure 4: The effects of monetary policy shock in DSGE-VAR model and DSGE model**

Note: The monetary policy shock (emp) – unexpected one-quarter 1 pp. increase in 3M Pribor (pribor). Responses: the exchange rate CZK/EUR (czkeur) – a decrease corresponds to appreciation; 3M Pribor (pribor), consumption deflator (defC), and private consumption (Cdiff).

---

6 The IRFs are reported with centered 68% of posterior distributions of the IRFs. Notice, however, that in this version of the paper, the matrix of impacts is taken as fixed. When we take draws of the matrix, the credible bands would be broader.
Now we turn to the discussion of the effects of fiscal shocks. The sizes of shocks are in all cases equal to one so they represent unexpected one percentage point changes in the quarter-on-quarter (QoQ) growth of the respective variable lasting for one quarter.

Regarding the shock to government consumption (eg), the IRFs based solely on impact matrix from the DSGE model are different from those implied by the combination of the VAR and DSGE impact matrices (Figure 5). The positive immediate reaction of private consumption on the increase in government consumption is in line with the Keynesian view that is inherently present in the DSGE model (the magnitude of the response depends inter alia on the share of rule-of-thumb households; this share is estimated and set to 16% according to the posterior mean). On the other hand, the reaction adjusted by the VAR exhibits a negative reaction on impact, which is in line with the neoclassical approach. Relaxing the assumptions of the DSGE model implies the move from Keynesian behaviour to neoclassical reactions.

Remarkable different reaction is also that of government investment. The DSGE model suggests that government investment reacts in the opposite direction to the government consumption shock. The DSGE-VAR model suggests the opposite. The magnitude of the reaction of the government investment is even higher than the magnitude of the original shock to government consumption.

**Figure 5: The effects of government consumption shock in DSGE-VAR model and DSGE model**

![Graph showing the effects of government consumption shock in DSGE-VAR and DSGE models](image)

*Note:* Government consumption spending shock (eg) – unexpected one-quarter 1 pp. increase in government consumption (Gdiff). Responses: government investment (Gldiff), private investment (Pldiff), private consumption (Cdiff), and government consumption (Gdiff).
Different IRFs between the DSGE and DSGE-VAR approaches can be observed also for the shock to government investment (Figure 6). Both private investment and private consumption suggest different signs of the impact of the shock. However, the profile of reaction of government consumption is very close in both approaches.

**Figure 6: The effects of government investment shock in DSGE-VAR model and DSGE model**

![Figure 6](image)

**Note:** Government investment shock ($e_{ig}$) – unexpected one-quarter 1 pp. increase in government investment ($G_{idiff}$). Responses: government investment ($G_{idiff}$), private investment ($P_{idiff}$), private consumption ($C_{diff}$), and government consumption ($G_{diff}$).
The effects of a positive other social benefits shock result in an opposite sign of responses of government consumption ($G_{diff}$): while in case of the DSGE model government consumption falls at impact, it raises in the DSGE-VAR framework. The responses of other variables are qualitatively comparable between the two modelling approaches (Figure 7): government investment ($G_{Idiff}$) falls at impact (although the DSGE response is characterised by large uncertainty), accompanied by a decline in private investment ($P_{Idiff}$) and consumption ($C_{diff}$); the variables then rise back toward the steady state.

**Figure 7: The effects of other social benefits shock in DSGE-VAR model and DSGE model**

![Figure 7: The effects of other social benefits shock in DSGE-VAR model and DSGE model](image)

*Note:* Other social benefits shock ($e_{ob}$) – unexpected one-quarter 1 pp. increase in other social benefits ($s_{bdiff}$). Responses: government investment ($G_{Idiff}$), private investment ($P_{Idiff}$), private consumption ($C_{diff}$), and government consumption ($G_{diff}$).
Finally, consumption tax and wage tax shock is discussed. Figure 8 presents the IRFs of the consumption tax shock defined as an unexpected increase of consumption tax by one percentage point. The IRFs from the DSGE model are in accordance with theory: an immediate fall in private consumption is observed. On the other hand, responses from the DSGE-VAR suggest no economically significant effect in reality (it is a consequence of almost no variation of the consumption tax rate in the data). Similar picture is obtained when labour tax shock is examined.

Figure 8: The effects of consumption tax shock in DSGE-VAR model and DSGE model

Note: Consumption tax shock (etc) – unexpected one-quarter 1 pp. increase in the consumption tax rate (tauC). Responses: consumption deflator (defC), private consumption (Cdiff), government consumption (Gdiff), and the consumption tax rate (tauC).

Difference in the IRFs between the DSGE and DSGE-VAR models suggests the presence of misspecification of the DSGE model. If there is no misspecification present, the IRFs between the two modelling approaches would coincide. In addition, some differences between the IRFs can be observed also for the monetary policy shocks – the IRFs in the DSGE model are standard, but the DSGE-VAR model exhibits some usual puzzles, e.g. the observed “price-puzzle”. So, the misspecification seems to be related not only to the fiscal components of the DSGE model.
7.2 Multipliers

Table 1 presents the cumulative fiscal multipliers at the horizon of 1, 4, 8 and 16 quarters. Several observations could be made. First, the DSGE-VAR framework results in somewhat larger multipliers than in the case of the DSGE block alone – for all three fiscal measures considered. The difference is particularly distinct for government consumption multipliers – DSGE-VAR cumulative multipliers (0.80–0.92) are about twice as large as those from the DSGE (0.42–0.47). The IRFs from Figure 5 show that the reason for the higher multipliers for DSGE-VAR is due to the government investment reaction.

Government investment multipliers are comparable at all horizons, taking the values in the range (0.18–0.20). Other social benefits multipliers are all negative, yet close to zero in the DSGE-VAR case starting from the 4th quarter, while exhibiting persistence in the DSGE model (-0.22 at the horizon of 16 quarters).

Table 1: Cumulative fiscal multipliers

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</table>

These differences in multipliers between the DSGE and DSGE-VAR framework are broadly in line with the evidence collected upon 104 studies examined in the meta analysis by Gechert (2015): DSGE models were found to result in lower multipliers, compared to the VAR estimates. The DSGE-VAR model, therefore, occupies the territory between the two modelling approaches.

<Comparison with the literature – to be completed>

8. Conclusions

<To be completed>
References


Appendix A: Marginal data density

Parameter \( \lambda \) determines how much parameters from the DSGE part of the DSGE-VAR model affect VAR parameters. It is obtained by maximizing the marginal data density

\[
p_\lambda (Y) = \int p_\lambda (Y | \theta)p(\theta)d\theta
\]

(A.1)

over the grid for \( \lambda \) covering non-negative values. Gelfand and Dey (1994) noted that

\[
\frac{1}{p_\lambda (Y)} = \int \frac{f(\theta)}{p_\lambda (Y | \theta)p(\theta)} p_\lambda (\theta | Y)d\theta = E\left[ \frac{f(\theta)}{p_\lambda (Y | \theta)p(\theta)} \right]
\]

(A.2)

because the posterior \( p_\lambda (\theta | Y) \) can be expressed as

\[
p_\lambda (\theta | Y) = \frac{p_\lambda (Y | \theta)p(\theta)}{p_\lambda (Y)}.
\]

(A.3)

Formula (A.2) suggests that we need not evaluate posterior \( p_\lambda (\theta | Y) \) to obtain marginal data density.

The inverse of marginal data density in (A.1) is then numerically approximated using Geweke’s (1999) modified harmonic mean estimator:

\[
\frac{1}{M} \sum_{j=1}^{M} \frac{f(\theta_j)}{p_\lambda (Y | \theta_j)p(\theta_j)}
\]

(A.4)

where \( \theta_j \) is a draw of DSGE parameter vector from the Metropolis Hastings algorithm and function \( f(\cdot) \) is defined as follows:

\[
f(\theta_j) = \frac{1}{p(2\pi)^{k/2} |\tilde{\Sigma}_\phi|^{1/2}} \exp \left[ -\frac{1}{2} (\theta_j - \tilde{\theta}) \tilde{\Sigma}_\phi^{-1}(\theta_j - \tilde{\theta})' \right] \times I(\theta_j)
\]

(A.5)

where \( \tilde{\theta} \) and \( \tilde{\Sigma}_\phi \) are posterior mean and covariance respectively, \( k \) denotes the number of parameters and the indicator function \( I(\theta_j) \) equals one if
\[
\left(\theta_j - \tilde{\theta} \right) \Sigma^{-1}_{j,j} \left(\theta_j - \tilde{\theta} \right) \leq \chi^2_{l-p}(k). \tag{A.6}
\]

\[\chi^2_{l-p}(k)\] is the inverse-chi-squared cumulative distribution function with \(k\) degrees of freedom and probability \(p\).

The likelihood function at a draw \(p_j(Y|\theta_j)\) is actually a marginal data density from the VAR part of the DSGE-VAR model:

\[
p_j(Y|\theta_j) = p_j(Y|\Phi, \Sigma)p_j(\Phi, \Sigma|\theta_j) / p_j(\Phi, \Sigma|Y) =
\]

\[
\frac{\left| \lambda T \Sigma^\dagger_x (\theta_j) + X X^T \right|^{-\frac{n}{2}} \left| (\lambda + 1) T \Sigma_a (\theta_j) \right|^{-\frac{n((\lambda+1)T-k)}{2}} \prod_{t=1}^{n} \Gamma \left[ \left( (\lambda + 1)T - k + 1 - i \right)/2 \right]}{\left| \lambda T \Sigma^\dagger_x (\theta_j) \right|^{-\frac{n}{2}} \left| \lambda T \Sigma_a (\theta_j) \right|^{-\frac{n(\lambda T-k)}{2}} \prod_{i=1}^{n} \Gamma \left[ \left( \lambda T - k + 1 - i \right)/2 \right]}.
\tag{A.7}
\]

The marginal likelihood is approximated using 200,000 draws from the MH algorithm.
## Appendix B: List of DSGE variables and structural shocks

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Units</th>
<th>ss¹</th>
<th>Shocks</th>
<th>Description</th>
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**Note:** ¹ steady-state values
Appendix C: Convergence Diagnostics

This appendix includes statistics suggesting convergence of the sampler used to estimate the DSGE-VAR model. Statistics employed are: autocorrelation of the chain at lag equal to 10, the inefficiency factor and Raftery and Lewis (1992) statistics showing the number of draws to get a stationary distribution for the Gibbs sampler. Figure C1 shows the statistics for all autoregressive parameters of the VAR model (2704 parameters), while Figure C2 displays the statistics for the draws of elements of the variance-covariance matrix of the VAR model in the first row of the matrix (26 parameters). All statistics demonstrates the convergence of the sampler.

Figure C1: All autoregressive parameters of the VAR

Figure C2: First raw parameters of the variance-covariance matrix of the VAR